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A Family of $p-$valent Analytic Functions with two Fixed Points Involving a Calculus Operator

By

MAMTA PATHAK AND SAHADAT ALI*

Abstract

In this paper a family $T_\lambda(m, \nu, \delta, \alpha, p, z_0)$ of $p-$valent analytic functions with two fixed points involving a calculus operator is considered. Necessary and sufficient coefficient condition for functions belonging to this family is obtained. Using this coefficient inequality, inclusion relation, growth theorem, extreme points, results on Hadamard product are obtained for functions belonging to this family.

1. Preliminaries

Let T(p) denotes a family of functions of the form

$$f(z) = |a_p|z^p - \sum_{k=1}^{\infty} |a_{p+k}|\, z^{p+k} \qquad (p \in N = \{1, 2, 3....\}), \tag{1}$$

which are analytic and $p-$valent in the open unit disk $U = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Denote $T(1) \equiv T$.

H. Silverman in 1976 [2] introduced and investigated subfamilies of starlike and convex functions of order α, consisting of functions $f(z) \in T$ with the requirement that:

$$\text{either } f(z_0) = z_0 \text{ or } f'(z_0) = 1 \text{ with } (-1 < \mathrm{z}_0 < 1; \mathrm{z}_0 \neq 0). \tag{2}$$

Uralegaddi and Somanatha [3] generalized the results by combining the requirements in (2) into a single condition. Murugusundaramoorthy [1] defined a family $T_\lambda(z_0)$ which is a subfamily of T with the requirement:

$$(1-\lambda)\frac{f(z_0)}{z_0} + \lambda f'(z_0) = 1(-1 < z_0 < 1, 0 \leq \lambda \leq 1). \tag{3}$$

Denote by $T_0(p, z_0)$ and $T_1(p, z_0)$ respectively subfamilies of functions $f(z) \in T(p)$ with the requirement that:either $z_0^{1-p} f(z_0) = z_0$ or $\frac{z_0^{1-p} f'(z_0)}{p} = 1$ with $(-1 < z_0 < 1; z_0 \neq 0)$.

*Corresponding author

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Denote by $T_\lambda(p, z_0)$ a subfamily of $T(p)$ with the requirement:

$$(1-\lambda)\frac{z_0^{1-p} f(z_0)}{z_0} + \lambda \frac{z_0^{1-p} f'(z_0)}{p} = 1 (-1 < z_0 < 1, 0 \leq \lambda \leq 1; z_0 \neq 0). \quad (4)$$

Using (1) and (4) it is obtained that,

$$|a_p| = 1 + \sum_{k=1}^{\infty} \frac{(p+\lambda k)}{p} |a_{p+k}| \, z_0^k. \quad (5)$$

Let $f(z) \in T_\lambda(p)$ be of the form (1) and $g(z)$ be of the form

$$g(z) = z^p - \sum_{k=1}^{\infty} |b_{p+k}| \, z^{p+k}, \quad (6)$$

then the Hadamard product of $f(z)$ and $g(z)$ is given by

$$(f * g)(z) = |a_p| z^p - \sum_{k=1}^{\infty} |a_{p+k}||b_{p+k}| \, z^{p+k}.$$

Let $S_p^*(\alpha)$ and $K_p(\alpha)$ respectively denote the families of $p-$valent starlike and convex functions in $T(p)$ which satisfy

$$\text{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \text{ and } \text{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, \qquad 0 \leq \alpha < p$$

respectively.

Definition 1.1. A normalized operator $\tilde{I}_p^{\delta,\nu} : A(p) \to A(p)$ for $\nu > -1-p$, $\delta + \nu > -1 - p$ is defined by

$$\tilde{I}_p^{\delta,\nu} f(z) = \frac{\Gamma(\nu+1+\delta+p)}{\Gamma(\nu+1+p)} \, z^{-\delta-\nu} I^{\delta,\nu} \, f(z). \quad (7)$$

The series expansion of $\tilde{I}_p^{\delta,\nu} f(z)$ is given by

$$\tilde{I}_p^{\delta,\nu} f(z) = z^p + \sum_{k=1}^{\infty} \theta_{p+k}(\delta,\nu) a_{p+k} z^{p+k}, \quad (8)$$

where for convenience

$$\theta_{p+k}(\delta,\nu) = \frac{(\nu+1+p)_k}{(\nu+1+\delta+p)_k}. \quad (9)$$

The symbol $(a)_k$ is called Pochhammer symbol defined as:

$$(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)} = a(a+1)...(a+k-1) \quad \text{and} \quad (a)_0 = 1.$$

Note that $\tilde{I}_p^{0,0} f(z) \equiv f(z)$, $\tilde{I}_p^{(}-1,0) f(z) \equiv \frac{zf'(z)}{p}$.

Definition 1.2. A function $f(z) \in T_\lambda(p, z_0)$ is said to be in $T_\lambda(m, \nu, \delta, \alpha, p, z_0)$ family if, it satisfies for $0 \leq \alpha < p$, $m \in N_0$, $\nu > -1-p$, $\delta + \nu > -1-p$

$$\mathrm{Re}\left\{ \frac{z(\tilde{I}_p^{\delta,\nu} f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu} f(z))^m} + m \right\} > \alpha. \tag{10}$$

Denote that $T_\lambda(0, \alpha, 0, 0, p, z_0) \equiv S^*_\lambda(\alpha, p, z_0)$ and $T_\lambda(0, \alpha, 0, -1, p, z_0) \equiv K_\lambda(\alpha, p, z_0)$.

2. Coefficient Inequality

In this section, necessary and sufficient coefficient condition for functions belonging to $T_\lambda(m, \nu, \delta, \alpha, p, z_0)$ family is obtained.

Theorem 2.1. Let $f(z) \in T_\lambda(p, z_0)$ of the form (1) satisfies

$$\sum_{k=1}^{\infty} \Psi_{p+k}(\alpha, \delta, \nu, m)|a_{p+k}| \leq |a_p|, \tag{11}$$

or, equivalently

$$\sum_{k=1}^{\infty} \left[\Psi_{p+k}(\alpha, \delta, \nu, m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k \right] |a_{p+k}| \leq (p-\alpha), \tag{12}$$

where

$$\Psi_{p+k}(\alpha, \delta, \nu, m) = \frac{(p+k)!(p-m)!(p+k-\alpha)}{(p+k-m)!\, p!} \theta_{p+k}(\delta, \nu), \tag{13}$$

for $0 \leq \alpha < p$, $\nu > -1-p$, $\delta+\nu > -1-p$, if and only if $f(z) \in T_\lambda(m, \nu, \delta, \alpha, p, z_0)$.

Proof. Let $f(z) \in T_\mu(m, \nu, \delta, \alpha, p, z_0)$, then from Definition 1.2, it gives

$$\mathrm{Re}\left\{ \frac{z(\tilde{I}_p^{\delta,\nu} f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu} f(z))^m} + m - \alpha \right\} > 0.$$

Taking z to be real it gives

$$\frac{p!}{(p-m-1)!}|a_p|z^{p-m} - \sum_{k=1}^{\infty} \frac{(p+k)!}{(p+k-m-1)!} \theta_{p+k}(\delta, \nu)|a_{p+k}|z^{p+k-m}$$

$$+(m-\alpha)\frac{p!}{(p-m)!}|a_p|z^{p-m} - \sum_{k=1}^{\infty} \frac{(p+k)!}{(p+k-m)!}(m-\alpha)\theta_{p+k}(\delta, \nu)|a_{p+k}|z^{p+k-m} \geq 0.$$

Letting $z \to 1^-$ along the real axis, it gives

$$(p-\beta)\frac{p!}{(p-m)!}|a_p| - \sum_{k=1}^{\infty} \frac{(p+k)!}{(p+k-m)!}(p+k-\alpha)\theta_{p+k}(\delta, \nu)|a_{p+k}| \geq 0$$

or,

$$\sum_{k=1}^{\infty}\frac{(p-m)!(p+k)!}{(p-\alpha)p!(p+k-m)!}(p+k-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}|\leq|a_p|,$$

on using (4), this proves (12).

Conversely, let (12) holds. It needs to show that (9) is satisfied and so $f(z)\in T_\lambda(m,\nu,\delta,\alpha,p,z_0)$. Using the fact that $\mathrm{Re}(w)>\alpha$ if and only if $|w-(1+\alpha)| < |w+(1-\alpha)|$, it is enough to show that,

$$\left|\frac{z(\tilde{I}_p^{\delta,\nu}f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu}f(z))^m}+m-(1+\alpha)\right|<\left|\frac{z(\tilde{I}_p^{\delta,\nu}f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu}f(z))^m}+m+(1-\alpha)\right|$$

Let

$$\begin{aligned}E&=\left|\frac{z(\tilde{I}_p^{\delta,\nu}f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu}f(z))^m}+m+(1-\alpha)\right|\\&=\left|(1+p-\alpha)\frac{p!}{(p-m)!}|a_p|z^{p-m}\right.\\&\qquad\left.-\sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-m)!}(1+p+k-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}|z^{p+k-m}\right|.\end{aligned}$$

Thus

$$E>(1+p-\alpha)\frac{p!}{(p-m)!}|a_p|-\sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-m)!}(1+p+k-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}|. \quad (14)$$

Again, let

$$\begin{aligned}F&=\left|\frac{z(\tilde{I}_p^{\delta,\nu}f(z))^{m+1}}{(\tilde{I}_p^{\delta,\nu}f(z))^m}+m-(1+\alpha)\right|\\&=\left|(p-1-\alpha)\frac{p!}{(p-m)!}|a_p|z^{p-m}\right.\\&\qquad\left.-\sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-m)!}(p+k-1-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}|z^{p+k-m}\right|.\end{aligned}$$

Thus

$$F<(1+\alpha-p)\frac{p!}{(p-m)!}|a_p|+\sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-m)!}(p+k-1-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}|. \quad (15)$$

Now, from (13), (14),

$$\begin{aligned} E-F &> (p-\alpha)\frac{p!}{(p-m)!}|a_p| + \sum_{k=1}^{\infty}\frac{(p+k)!}{(p+k-m)!}(p+k-\alpha)\theta_{p+k}(\delta,\nu)|a_{p+k}| \\ &= (p-\alpha) - \sum_{k=1}^{\infty}\left[\frac{(p-m)!(p+k)!(p+k-\alpha)}{(p+k-m)!\,p!}\theta_{p+k}(\delta,\nu)\right. \\ &\quad \left. -\frac{(p-\alpha)(p+\lambda k)}{p}z_0^k\right]|a_{p+k}| \\ &> 0, \text{if (12) holds.} \end{aligned}$$

This completes the proof. The result is sharp for the function:

$$f_k(z) = |a_p|z^p - \frac{(p-\alpha)}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p}z_0^k\right)}z^{p+k}, \quad k \geq 1.$$

Corollary 2.2. Let $f(z) \in T_\mu(m,\nu,\delta,\alpha,p,z_0)$, then for $0 \leq \alpha < p$, $\nu > -1-p$, $\delta+\nu > -1-p$,

$$|a_{p+k}| \leq \frac{(p-\alpha)}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p}z_0^k\right)}, \quad k \geq 1.$$

For $m=\delta=\nu=0$ and $m=0$, $\delta=-1$, $\nu=0$ respectively in Theorem 2.1, following results are obtained.

Corollary 2.3. Let $f(z) \in T_\lambda(p,z_0)$ of the form (1) satisfies

$$\sum_{k=1}^{\infty}\left[\frac{(p+k-\alpha)}{(p-\alpha)} - \frac{(p+\lambda k)}{p}z_0^k\right]|a_{p+k}| \leq 1,$$

for $0 \leq \alpha < p$, $\nu > -1-p$, $\delta+\nu > -1-p$, if and only if $f(z) \in S^*_\lambda(\alpha,p,z_0)$.

Corollary 2.4. Let $f(z) \in T_\lambda(p,z_0)$ of the form (1) satisfies

$$\sum_{k=1}^{\infty}\left[\frac{(p+k-\alpha)(p+k)}{p} - \frac{(p-\alpha)(p+\lambda k)}{p}z_0^k\right]|a_{p+k}| \leq (p-\alpha),$$

for $0 \leq \alpha < p$, $\nu > -1-p$, $\delta+\nu > -1-p$, if and only if $f(z) \in K_\lambda(\beta,p,z_0)$.

Further, a consequence result of Theorem 2.1 is given in the form of following corollary.

Corollary 2.5. Let $f(z) \in T_\lambda(p,z_0)$ of the form (1) and for $m \in N_0$, $0 \leq \alpha < p$, $\nu > -1-p$, $\delta+\nu > -1-p$. Then

$$T_\lambda(m+1,\nu,\delta,\alpha,p,z_0) \subset T_\lambda(m,\nu,\delta,\alpha,p,z_0).$$

Proof. Since $\frac{(p-m)!}{(p+k-m)!}$ is an increasing function of m. Hence, using coefficient inequality (12),the result can be easily proved.

3. Growth Theorem

In this section, growth result of $f(z) \in T_\lambda(m, \nu, \delta, \alpha, p, z_0)$ family is obtained.

Theorem 3.1. Let $f(z) \in T_\lambda(p, z_0)$ of the form (2) be in the family $S^*_\lambda(\alpha, p, z_0)$, then for $|z| = r < 1$,

$$|\tilde{I}_p^{\delta,\nu} f(z)| \leq |a_p| \left(r^p + \frac{(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)} r^{p+1} \right) \tag{16}$$

$$|\tilde{I}_p^{\delta,\nu} f(z)| \geq |a_p| \left(r^p - \frac{(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)} r^{p+1} \right). \tag{17}$$

Proof. In view of inequality (11),

$$\frac{(p+1)!(p-m)!(p+1-\alpha)}{(p+1-m)!\,p!} \sum_{k=1}^{\infty} \theta_{p+k}(\delta,\nu)|a_{p+k}|$$
$$\leq \sum_{k=1}^{\infty} \frac{(p+k)!(p-m)!(p+k-\alpha)}{(p+k-m)!\,p!} \theta_{p+k}(\delta,\nu)|a_{p+k}| \leq |a_p|(p-\alpha)$$

which gives

$$\sum_{k=1}^{\infty} \theta_{p+k}(\delta,\nu)|a_{p+k}| \leq \frac{|a_p|(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)}.$$

Therefore,

$$|\tilde{I}_p^{\delta,\nu} f(z)| \leq |a_p| r^p + \frac{|a_p|(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)} r^{p+1}$$

and

$$|\tilde{I}_p^{\delta,\nu} f(z)| \geq |a_p| r^p - \frac{|a_p|(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)} r^{p+1},$$

where $|a_p|$ is given by (5). This proves (16) and (17).

Corollary 3.2. Let $f(z) \in T_\lambda(m, \nu, \delta, \alpha, p, z_0)$. Then $\tilde{I}_p^{\delta,\nu} f(z)$ is included in a disk with its centre at the origin and radius R given by

$$R = |a_p| \left(1 - \frac{(p-\alpha)(p+1-m)}{(p+1)(p+1-\alpha)} \right).$$

4. Extreme Points

Theorem 4.1. Let $f_0(z) = |a_p| z^p$ and

$$f_k(z) = |a_p| z^p - \frac{(p-\alpha)}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right)} z^{p+k}, \quad k \geq 1, \tag{18}$$

then $f(z) \in T_\lambda(m,\nu,\delta,\alpha,p,z_0)$, if and only if it can be expressed in the form $f(z) = \sum\limits_{k=0}^{\infty} d_k f_k(z)$, where $d_k \geq 0$ and $\sum\limits_{k=0}^{\infty} d_k = 1$.

Proof. Suppose

$$f(z) = \sum_{k=0}^{\infty} d_k f_k(z) = |a_p| z^p - \sum_{k=1}^{\infty} \frac{(p-\alpha) d_k}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right)} z^{p+k}.$$

Then

$$\sum_{k=1}^{\infty} \left[\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right] \frac{(p-\alpha) d_k}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right)}$$
$$= \sum_{k=1}^{\infty} (p-\alpha) d_k \leq \sum_{k=0}^{\infty} (p-\alpha) d_k$$
$$= (p-\alpha)(1-d_0) \leq (p-\alpha).$$

Hence, from (11), $f(z) \in T_\lambda(m,\nu,\delta,\alpha,p,z_0)$.

Conversely, suppose that $f(z) \in T_\lambda(m,\nu,\delta,\alpha,p,z_0)$. Corollary 2.2 gives

$$|a_{p+k}| \leq \frac{(p-\alpha)}{\left(\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right)}, \quad k \geq 1.$$

Setting

$$d_k = \left[\Psi_{p+k}(\alpha,\delta,\nu,m) - \frac{(p-\alpha)(p+\lambda k)}{p} z_0^k\right] |a_{p+k}|, \quad k \geq 1$$

and

$$1 - \sum_{k=1}^{\infty} d_k = d_0.$$

On using (18), it is seen that

$$f(z) = \sum_{k=0}^{\infty} d_k f_k(z).$$

5. Results on Modified Hadamard Product

Theorem 5.1. If $f(z)$ of the form (1) belongs to $T_\lambda(m,\nu,\delta,\alpha,p,z_0)$ and $g(z) = z^p - \sum\limits_{k=1}^{\infty} |b_{p+k}| z^{p+k} \in T_\lambda(m,\nu,\delta,\zeta,p,z_0)$, then $(f*g)(z) \in T_\lambda(m,\nu,\delta,\varsigma,p,z_0)$, where

$$\varsigma = \min_{k\geq 1} \left[p - \frac{(p-\alpha)(p-\zeta)|a_p|}{\Psi_{p+k}(\alpha,\delta,\nu,m)(p+k-\zeta) - (p-\alpha)(p-\zeta)|a_p|}\right] \tag{19}$$

and $\Psi_{p+k}(\alpha,\delta,\nu,m)$ is given by (13).

Proof. In view of Theorem 2.1, it is sufficient to show that,

$$\sum_{k=1}^{\infty}\frac{(p+k)!(p-m)!(p+k-\varsigma)}{(p-\varsigma)(p+k-m)!\,p!\,|a_p|}\theta_{p+k}(\delta,\nu)|a_{p+k}||b_{p+k}|\le 1$$

for ς defined in (19). Now, if $f(z)\in T_\lambda(m,\nu,\delta,\alpha,p,z_0)$ and $g(z)\in T_\lambda(m,\nu,\delta,\zeta,p,z_0)$,

$$\sum_{k=1}^{\infty}\frac{(p+k)!(p-m)!(p+k-\alpha)}{(p-\alpha)(p+k-m)!\,p!\,|a_p|}\theta_{p+k}(\delta,\nu)|a_{p+k}|\le 1 \tag{20}$$

$$\sum_{k=1}^{\infty}\frac{(p+k)!(p-m)!(p+k-\zeta)}{(p-\zeta)(p+k-m)!\,p!\,|a_p|}\theta_{p+k}(\delta,\nu)|b_{p+k}|\le 1. \tag{21}$$

On applying Cauchy-Schwartz inequality to (20) and (21),

$$\sum_{k=1}^{\infty}\frac{(p+k)!(p-m)!\sqrt{(p+k-\zeta)(p+k-\alpha)}}{\sqrt{(p-\alpha)(p-\zeta)}(p+k-m)!p!|a_p|}\theta_{p+k}(\delta,\nu)\sqrt{|a_{p+k}||b_{p+k}|}\le 1. \tag{22}$$

Thus (22) holds true, if

$$\sqrt{|a_{p+k}||b_(p+k)|}\le\frac{\sqrt{(p+k-\zeta)(p+k-\alpha)}(p-\varsigma)}{\sqrt{(p-\alpha)(p-\zeta)}(p+k-\varsigma)}. \tag{23}$$

In view of (22) and (23), it is sufficient to show that

$$\frac{\sqrt{(p-\alpha)(p-\zeta)}(p+k-m)!p!|a_p|}{(p+k)!(p-m)!\sqrt{(p+k-\zeta)(p+k-\alpha)}\theta_{p+k}(\delta,\nu)}$$
$$\le\frac{\sqrt{(p+k-\zeta)(p+k-\alpha)}(p-\varsigma)}{\sqrt{(p-\alpha)(p-\zeta)}(p+k-\varsigma)}$$

or,

$$\varsigma=\min_{k\ge 1}\left[p-\frac{(p-\alpha)(p-\zeta)|a_p|}{\Psi_{p+k}(\alpha,\delta,\nu,m)(p+k-\zeta)-(p-\alpha)(p-\zeta)|a_p|}\right],$$

where $|a_p|$ is given by (5). This completes the proof.

For $\zeta=\alpha$ in Theorem 5.1, following corollary is obtained.

Corollary 5.2. If $f(z)$ of the form (1) and $g(z)=z^p-\sum_{k=1}^{\infty}|b_{p+k}|z^{p+k}$, both are in $T_\lambda(m,\nu,\delta,\alpha,p,z_0)$, then $(f*g)(z)\in T_\lambda(m,\nu,\delta,\varsigma,p,z_0)$, where

$$\varsigma=\min_{k\ge 1}\left[p-\frac{(p-\alpha)^2|a_p|}{\Psi_{p+k}(\alpha,\delta,\nu,m)(p+k-\alpha)-(p-\alpha)^2|a_p|}\right].$$

Shri Ramswroop Memorial Group of Professional Colleges,
Lucknow, India
E-mail: pathakmamta_2007@rediffmail.com, sdalilko@rediffmail.com

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