J. Nat. Acad. Math. ISSN 0970-5228
Vol. 35 (2021), pp.14-23

Applications of Multivariable *H*-Function of Srivastava-Panda and Generalized Polynomials of Srivastava in a Problem on Heat Conduction

By

S.S. Chauhan* and R.C. Singh Chandel

Department of Mathematics, D.V. College, Orai (Jalaun), UP

Email: dr.surendrasingh2010@gmail.com

Abstract
In the present paper, First we evaluate an integral involving the product of multivariable *H*-function of Srivastava and Panda ([30],[31],[32]), several generalized polynomials of Srivastava [25], a generalizations of multivariable polynomials of Chandel and Tiwari [10] and Hermite Polynomials; and then we make its applications in solving a problem on heat conduction given by Bhonsle [1]. One expansion formula is also established. Finally, we also discuss special cases for different polynomials.

Keywords: Hermite polynomials, Jacobi polynomials, Lebdve equation
2020 Mathematics Subject Classification: 33C50

1. Introduction

Appell's functions and the functions related to them have many applications in mathematical physics ([19],[20],[21]), Srivastava, Gupta and Goyal [33]) have discussed a problem on heat conduction in a finite bar using *H*-function of two variables of Srivastava and Panda ([30],[31],[32]). Singh [23] used generalized hypergeometric function in a problem of cooling of a heated cylinder. Further Singh [24] evaluated some integrals involving Kampe de Feriet function and one of them was employed to obtain a solution of a problem on heat conduction given by Bhonsle [1]. Chandel and Yadava [3] have evlauted certain integrals involving multiple hypergeometric function of Srivastava and Daoust ([26],[27],[28]); also see Srivastava and Karlsson [29, p.37, eqns. (2.1) to (2.3)], and their applications have been given in solving the same problem on heat conduction. Chandel-Bhargava [2] have used generalized Kampe de Feriet function of two variables due to Srivastava-Daoust ([26],[27],[28]), while Chandel-Gupta [5], have used multivariable *H*-function of Srivastava and Panda ([30],[31],[32]; also see Srivastava Gupta and Goyal [33]) in a problem of colling of a heated cylinder. Chandel and Gupta [4] have also used *H*-function of several variables in a problem of heat conduction. Chandel and Tiwari [6] employed multiple hypergeometric function of several variables due to Srivastava and Daoust ([26],[27],[28]) in two boundary value problems. Chaurasia and Patni [14] have discussed a heat conduction problem involving the product of multivariable *H*-function and two general classes of polynomials, while Chaurasia and Gupta [15] have discussed a solution of partial differential equation of heat conduction in a rod under Robin condition.

Recently Chandel and Sengar [7] have discussed two boundary value problems on heat conduction involving the product of multivariable *H*-function of Srivastava-Panda ([30],[31],[32]) and several generalized polynomials of Srivastava [25] and their special cases have been discussed. Further

Received: April 14, 2021; Accepted: August 24, 2021

Chandel and Sengar [8] have discussed a problem on heat conduction in a rod under the Robin condition involving the product of above multivariable *H*-function and several generalized polynomials of Srivastava [25} and a generalization of multivariable polynomials of Chandel and Tiwari [10].

Chandel and Sengar [11] discussed multivariable generalized polynomials defined through their generating function. Also, Chandel and Sharma [12] discussed a multivariable analogue of a class polynomials. Further Chandel and Kumar [13] discussed a contour integral representation of two variable generalized hypergeometric function of Srivastava and Daoust. Also, Kumar and Rai [18] discussed multiple fractional diffusions via multivariable *H*-function.

First, we evaluate an integral involving the product of multivariable *H*-function of Srivastava and Panda ([30],[31],[32]), several generalized polynomials of Srivastava [25] and Hermite polynomials; and then we make its applications in solving a problem on heat conduction given by Bhonsle [1]. One expansion formula is also established. Finally, we also discuss special cases for different polynomials.

2. Main Integral

In this section, we evaluate the integral

$$\int_{-\infty}^{\infty} z^{2\rho} e^{-z^2} H_{2v}(z) H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: \\ [(c):\Psi',\dots,\Psi^{(n)}]:\end{matrix}\right. \tag{2.1}$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} x_1 z^{2\alpha_1},\dots,x_n z^{2\alpha_n}\right) \prod_{i=1}^{r} S_{n_i}^{m_i}\left[y_i z^{2\beta_i}\right] dz$$

$$= \sqrt{\pi}\, 2^{2(v-p)} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i} A_{n_i k_i}}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}], \\ [(c):\Psi',\dots,\Psi^{(n)}],\end{matrix}\right.$$

$$\left.\begin{matrix}[-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [\nu-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,2\alpha_n]: \;[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} \frac{x_1}{4^{\alpha_i}},\dots,\frac{x_n}{4^{\alpha_n}}\right).$$

where, $H_{2v}(z)$ are Hermite polynomials (see Rainville [22]). $S_n^m[z]$ are generalized polynomials of Srivastava [25], $H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}$ is multivariable *H*-function of Srivastava-Panda ([30],[31],[32]; also see Srivastava, Gupta and Goyal [33]);

$$|arg\, x_i z^{2\alpha_i}| < \frac{\pi}{2}\Delta_i,$$

$$\Delta_i = \sum_{j=1}^{\lambda}\theta_j^{(i)} - \sum_{j=\lambda+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{\nu^{(i)}}\phi_j^{(i)} - \sum_{j=\nu^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{\mu'}\delta_j^{(i)}$$

$$-\sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0,1,\dots,n$$

$\rho = 0,1,2,\dots; 2v, n_j, m_j$ are positive integers and the coefficients A_{n_j,k_j} (j=1,…,r) are arbitrary parameters real or complex independent of $y_1,\dots,y_r,z$.

This integral will be quite useful in our further investigations.

Proof. Multiplying both sides of Lebdev equation [20, (4.16.1)] by $e^{-z^2}H_{2v}(z)$ and using orthogonal property of Hermite polynomials [22], we have

$$\int_{-\infty}^{\infty} z^{2\rho}e^{-z^2}\,H_{2v}(z)dz=\frac{\sqrt{\pi}2^{2(v-\rho)}\Gamma(2\rho+1)}{\Gamma(\rho-v+1)},\rho=0,1,2,\dots. \tag{2.2}$$

Now left hand side of (2.1)

$$=\sum_{k_i=0}^{[n_1,m_1]}\cdots\sum_{k_r=0}^{[n_r,m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_ik_i}A_{n_ik_i}y_i^{k_i}}{k_i!}\frac{1}{(2\pi\omega)^n}\int_{L_1}\cdots\int_{L_n}\prod_{i=1}^{n}\frac{\prod_{j=1}^{\mu^{(i)}}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j+\mu^{(i)}=1}^{D^{(i)}}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)}$$

$$\frac{\prod_{j=1}^{v^{(i)}}\Gamma\left(1-b_j^{(i)}+\phi_j^{(i)}s_i\right)\prod_{j=1}^{\lambda}\Gamma\left(1-a_j^{(i)}+\theta_j^{(i)}s_i\right)}{\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma\left(b_j^{(i)}-\phi_j^{(i)}s_j\right)\prod_{j=\lambda+1}^{A}\Gamma\left(a_j-\sum_{i=1}^{n}\theta_j^{(i)}s_j\right)\prod_{j=1}^{C}\Gamma\left(1-c_j-\sum_{i=1}^{n}\psi_j^{(i)}s_i\right)}$$

$$\left(\int_{-\infty}^{\infty}e^{-z^2}\,H_{2v}(z)z^{2\rho+2\beta_1k_1+\cdots+2\beta_rk_r+2\alpha_1s_1+\cdots+2\alpha_ns_n}dz\right)ds_1\dots ds_n$$

= right hand side of (2.1) (By an appeal to (2.2)

3. Application of Heat Conduction.

Bhonsle [1] has employed Hermite polynomials in solving the partial differential equation

$$\frac{\partial\phi}{\partial t}=K\frac{\partial^2\phi}{\partial z^2}-K\phi z^2\,, \tag{3.1}$$

where $\phi(z,t)$ tends to zero for a large value of *t* and when $|z|\to\infty$, this equation is related to the problem of heat conduction due to Churchill [16].

$$\frac{\partial\phi}{\partial t}=K\frac{\partial^2\phi}{\partial z^2}h_1(\phi-\phi_0), \tag{3.2}$$

provided that $\phi_0=0$ and $h_1=Kz^2$.

The solution of (3.1) given by Bhonsle [1] is

$$\phi(z,t)=\sum_{s=0}^{\infty}A_se^{-(1+2s)Kt-\frac{z^2}{2}}H_s(z) \tag{3.3}$$

Here we consider the problem of determining $\phi(z,t)$, where for *t* = 0.

$$\phi(z,0)=f(z)=z^{2\rho}e^{-z^2}H^{0,\lambda:(\mu',v');\dots;(\mu^{(n)},v^{(n)})}_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:\\ [(c):\Psi',\dots,\Psi^{(n)}]:\end{matrix}\right. \tag{3.4}$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}x_1z^{2\alpha_1},\dots,x_nz^{2\alpha_n}\right)\prod_{i=1}^{r}S_{n_i}^{m_i}[y_iz^{2\beta_i}]$$

Thus by (3.3) and (3.4), we have

$$\int_{-\infty}^{\infty}e^{-z^2}\,H_{2v}(z)H^{0,\lambda:(\mu',v');\dots;(\mu^{(n)},v^{(n)})}_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:\\ [(c):\Psi',\dots,\Psi^{(n)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];\\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}x_1z^{2\alpha_1},\dots,x_nz^{2\alpha_n}\right)\prod_{i=1}^{r}S_{n_i}^{m_i}[y_iz^{2\beta_i}]\mathrm{dz}$$

$$= \sum_{s=0}^{\infty} A_s \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} H_s(z) H_{2\nu}(z) dz$$

$= \sqrt{2\pi}(2\nu)!\, A_{2\nu}$ (by orthogonal property of Hermite polynomials

Erdelyi (17,p.289]).

Therefore

$$A_s = \frac{2^{s-2\rho-1/2}}{s!} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i} A_{n_i k_i}}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}} H^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})} \left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}], \ [-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]; \\ [(c):\Psi',\dots,\Psi^{(n)}], \ \left[\frac{s}{2}-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,\alpha_n\right]; \end{matrix} \right. \left. \begin{matrix} [(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}} \right), \tag{3.5}$$

where all conditions of (2.1) are satisfied.

Thus, substituting the value of A_S from (3.5) in (3.3), the solution of the main problem is given by

$$\phi(z,t) = \frac{e^{-\frac{z^2}{2}}}{2^{(4\rho+1)/2}} \sum_{s=0}^{\infty} H_s(z) e^{-(1+2s)k_1} \frac{2^s}{s!} \sum_{k_i=0}^{[n_1,m_1]} \dots \sum_{k_r=0}^{[n_r,m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i} A_{n_i k_i}}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}} \tag{3.6}$$

$$H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})} \left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}], \ [-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]: \\ [(c):\Psi',\dots,\Psi^{(n)}], \left[-\frac{s}{2}-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,2\alpha_n\right]: \end{matrix} \right. \left. \begin{matrix} [(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}} \right),$$

provided that all conditions of (2.1) are satisfied.

4. Expansion Formula.

An appeal to (3.4) and (3.6) gives the following expansion formula

$$2^{(4\rho+1)/2} z^{2\rho} e^{-z^2/2} H^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})} \left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}]; \\ [(c):\Psi',\dots,\Psi^{(n)}]; \end{matrix} \right. \left. \begin{matrix} [(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} x_1 z^{2\alpha_1},\dots,x_n z^{2\alpha_n} \right) \prod_{i=1}^{r} S_{n_i}^{m_i}[y_i z^{2\beta_i}] \tag{4.1}$$

$$= \sum_{s=0}^{\infty} H_s(Z) \frac{2^s}{s!} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i} A_{n_i k_i}}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}} H^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})} \left(\begin{matrix} [(a):\theta',\dots,\theta^{(n)}]: \ [-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]; \\ [(c):\Psi',\dots,\Psi^{(n)}]: \ \left[-\frac{s}{2}-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,\alpha_n\right]; \end{matrix} \right. \left. \begin{matrix} [(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]; \end{matrix} \frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}} \right),$$

valid if all conditions (2.1) are satisfied:

5. Special Cases.

Case I. For each $m_i = 2,\ A_{n_i,k_i} = (-1)^{k_i}$, we have

$$S_{n_i}^2[y_i] \to y_i^{n_i} H_{n_i}\left(\frac{1}{2\sqrt{y_i}}\right), i = 1, \dots, r.$$

Therefore, for Hermite polynomials ([35], p. 106, equation (5.5.4) and [34], p. 158) our main integral (2.1) reduces to

$$\int_{-\infty}^{\infty} z^{2\rho} e^{-z^2} H_{2v}(z) \prod_{i=1}^{r} \left(y_i z^{2\beta_1}\right)^{\frac{n_i}{2}} H_{n_i}\left(\frac{1}{2\sqrt{y_i} z^{\beta_i}}\right) H_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})} \quad (5.1)$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: [(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\Psi',\dots,\Psi^{(n)}]: [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} x_1 z^{2\alpha_1},\dots,x_n z^{2\alpha_n}\right) dz$$

$$= \sqrt{\pi} 2^{2(\nu-\rho)} \sum_{k_1=0}^{n_1/2} \dots \sum_{k_r=0}^{n_r/2} \prod_{i=1}^{r} \frac{(-n_i)_{2k_i}(-1)^k}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]; [-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]: \\ [(c):\Psi',\dots,\Psi^{(n)}]; \quad [\nu-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,2\alpha_n]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} \frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}}\right),$$

where all conditions of (2.1) are satisfied.

Thus solution (3.6) of the problem reduces to

$$\phi(z,t) = \frac{1}{2^{(4\rho+1)^2}} e^{-z^2/2} \sum_{s=0}^{\infty} \frac{2^s}{s!} e^{-(1+2s)K_i} \quad (5.2)$$

$$H_s(z) \sum_{k_1=0}^{n_1/2} \dots \sum_{k_r=0}^{n_r/2} \prod_{i=1}^{r} \frac{(-n_i)_{2k_i}(-1)^k}{k_i!} \frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}], [-2\rho-2\beta_1 k_1-\dots-2\beta_r k_r: 2\alpha_1,\dots,2\alpha_n]: \\ [(c):\Psi',\dots,\Psi^{(n)}], \left[-\frac{s}{2}-\rho-\beta_1 k_1-\dots-\beta_r k_r: \alpha_1,\dots,\alpha_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} \frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}}\right),$$

provided that all conditions of (2.1) are satisfied.

The expansion formula (4.1) reduces to

$$2^{(4\rho+1)/2} z^{2\rho} e^{-z^2} H_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: \\ [(c):\Psi',\dots,\Psi^{(n)}]:\end{matrix}\right. \quad (5.3)$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} x_1 z^{2\alpha_1},\dots,x_n z^{2\alpha_n}\right) \prod_{i=1}^{r} y_i^{n_i/2} z_i^{\beta_i n_i} H_{n_i}\left(\frac{1}{2\sqrt{y_i} z^{\beta_i}}\right)$$

$$= \sum_{s=0}^{\infty} \frac{2^s}{s!} H_s(Z) \sum_{k_1=0}^{n_1/2} \cdots \sum_{k_r=0}^{n_r/2} \prod_{i=1}^{r} \frac{(-n_i)_{k_i}}{k_i!} \frac{(1)^{k_i}\, y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: & [-2\rho-2\beta_1 k_1-\cdots-2\beta_r k_r: 2\alpha_1,\ldots,2\alpha_n]: \\ [(c):\Psi',\ldots,\Psi^{(n)}]: & \left[\frac{s}{2}-\rho-\beta_1 k_1-\cdots-\beta_r k_r:\alpha_1,\ldots,\alpha_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right]; \\ [(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ \frac{x_1}{4^{\alpha_1}},\ldots,\frac{x_n}{4^{\alpha_n}}\right),$$

valid if all conditions of (2.1) are satisfied.

Case II. For each $m_i = 1, A_{n_i k_i} = \binom{n_i+\gamma_i}{n_i}\frac{1}{(1+\gamma_i)_{k_i}}$,

we have $S^1_{n_i}[y_i] \to L^{(\gamma_i)}_{n_i}(y_i);\ \ i = 1,\ldots,r$

Therefore, for Laguerre polynomials ([35], p.10, eqn. (5.1.6) and [34], p. 158), the main integral (2.1) reduces to

$$\int_{-1}^{1} z^{2\rho} e^{-z^2} H_{2v}(z) L^{(\gamma_i)}_{n_i}\left(y_i z^{2\beta_i}\right) H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:(B',D');\ldots;(B^{(n)},D^{(n)})} \tag{5.4}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:\ [(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right]; \\ [(c):\Psi',\ldots,\Psi^{(n)}]:\ [(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ x_1 z^{2\alpha_1},\ldots,x_n z^{2\alpha_n}\right) dz$$

$$= \sqrt{\pi}\, 2^{2(\nu-\rho)} \sum_{k_1=0}^{n_1} \cdots \sum_{k_r=0}^{n_r} \prod_{i=1}^{r} \frac{(-n_i)_{k_i}}{k_i!} \binom{n_i+\gamma_i}{n_i} \frac{1}{(1+\gamma_i)_{k_i}} \frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: & [-2\rho-2\beta_1 k_1-\cdots-2\beta_r k_r: 2\alpha_1,\ldots,2\alpha_n]: \\ [(c):\Psi',\ldots,\Psi^{(n)}]: & [\nu-\rho-\beta_1 k_1-\cdots-\beta_r k_r:\alpha_1,\ldots,\alpha_n]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right]; \\ [(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ \frac{x_1}{4^{\alpha_1}},\ldots,\frac{x_n}{4^{\alpha_n}}\right),$$

provided that all conditions (2.1) are satisfied.

Therefore, solution (3.6) of the problem reduces to

$$\phi(z,t) = \sum_{s=0}^{\infty} \frac{2^{s-2\rho-1/2}}{s!} e^{-(1+2s)Kt-z^2/2} \tag{5.5}$$

$$H_s(z) \sum_{k_1=0}^{n_1} \cdots \sum_{k_r=0}^{n_r} \prod_{i=1}^{r} \frac{(-n_i)_{k_i}}{k_i!} \binom{n_i+\gamma_i}{n_i} \frac{1}{(1+\gamma_i)_{k_i}} \frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: & [-2\rho-2\beta_1 k_1-\cdots-2\beta_r k_r: 2\alpha_1,\ldots,2\alpha_n]: \\ [(c):\Psi',\ldots,\Psi^{(n)}]: & \left[\frac{s}{2}-\rho-\beta_1 k_1-\cdots-\beta_r k_r:\alpha_1,\ldots,\alpha_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right]; \\ [(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ \frac{x_1}{4^{\alpha_i}},\ldots,\frac{x_n}{4^{\alpha_n}}\right),$$

valid if all conditions of (3.1) are satisfied.

The expansion formula (4.1) reduces to

$$2^{2\rho+1/2}z^{2\rho}e^{-z^2}\prod_{i=1}^{r}L_{n_i}^{(\gamma_i)}\left(y_iz^{2\beta_i}\right)H_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: \\ [(c):\Psi',\dots,\Psi^{(n)}]:\end{matrix}\right. \tag{5.6}$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix} x_1z^{2\alpha_1},\dots,x_nz^{2\alpha_n}\right)$$

$$=\sum_{s=0}^{\infty}H_s(Z)\frac{2^s}{s!}\sum_{k_1=0}^{n_1}\dots\sum_{k_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{k_i}}{k_i!}\binom{n_i+\gamma_i}{n_i}\frac{1}{(1+\gamma_i)_{k_i}}\frac{y_i^{k_i}}{4^{\beta_ik_i}}$$

$$H_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: & [-2\rho-2\beta_1k_1-\dots-2\beta_rk_r:2\alpha_1,\dots,2\alpha_n]: \\ [(c):\Psi',\dots,\Psi^{(n)}]: & \left[\frac{s}{2}-\rho-\beta_1k_1-\dots-\beta_rk_r:\alpha_1,\dots,\alpha_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\frac{x_1}{4^{\alpha_1}},\dots,\frac{x_n}{4^{\alpha_n}}\right),$$

provided that all conditions of (2.1) are satisfied.

Case III For each $m_i=1, A_{n_ik_i}=\binom{n_i+\xi_i}{n_i}\frac{(1+\xi_i+\eta_i+n_i)}{(1+\xi_i)_{k_i}}$,

we have $S_{n_i}^1[y_i]\rightarrow P_{n_i}^{(\xi_i,\eta_i)}(1-2y_i)$,

Therefore, for Jacobi polynomials ([35], p.68, eq. (4.3.2) and [34], p. 159), (2.1) reduces to

$$\int_{-\infty}^{\infty}z^{2\rho}e^{-z^2}H_{2v}(z)\prod_{i=1}^{r}P_{n_i}^{(\xi_i,\eta_i)}\left(1-2y_iz^{2\beta_i}\right)H_{A,C:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})} \tag{5.7}$$

$$\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(c):\Psi',\dots,\Psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}x_1z^{2\alpha_1},\dots,x_nz^{2\alpha_n}\right)$$

$$=\sqrt{\pi}2^{2(\nu-\rho)}\sum_{k_1=0}^{n_1}\dots\sum_{k_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{k_i}}{k_i!}\binom{n_i+\xi_i}{n_i}\frac{(1+\xi_i+\eta_i+n_i)}{(1+\xi_i)_{k_i}}\frac{y_i^{k_i}}{4^{\beta_ik_i}}$$

$$H_{A+1,C+1:(B',D');\dots;(B^{(n)},D^{(n)})}^{0,\lambda+1:(\mu',\nu');\dots;(\mu^{(n)},\nu^{(n)})}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]: & [-2\rho-2\beta_1k_1-\dots-2\beta_rk_r:2\alpha_1,\dots,2\alpha_n]: \\ [(c):\Psi',\dots,\Psi^{(n)}]: & [\nu-\rho-\beta_1k_1-\dots-\beta_rk_r:\alpha_1,\dots,\alpha_n]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\frac{x_1}{4^{\alpha_i}},\dots,\frac{x_n}{4^{\alpha_n}}\right),$$

provided that all conditions (2.1) are satisfied.

Therefore, solution (3.6) of the problem reduces to

$$\phi(z,t)=\sum_{s=0}^{\infty}H_s(z)e^{-(1+2s)Kt-z^2/2} \tag{5.8}$$

$$\sum_{k_1=0}^{n_1}\cdots\sum_{k_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{k_i}}{k_i!}\binom{n_i+\xi_i}{n_i}\frac{(1+\xi_i+\eta_i+n_i)}{(1+\xi_i)_{k_i}}\frac{y_i^{k_i}}{4^{\beta_i k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: & [-2\rho-2\beta_1k_1-\cdots-2\beta_rk_r:2\alpha_1,\ldots,2\alpha_n]: \\ [(c):\Psi',\ldots,\Psi^{(n)}]: & \left[\frac{s}{2}-\rho-\beta_1k_1-\cdots-\beta_rk_r:\alpha_1,\ldots,\alpha_n\right]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\frac{x_1}{4^{\alpha_i}},\ldots,\frac{x_n}{4^{\alpha_n}}\right),$$

valid if all conditions of (2.1) are satisfied.

The expansion formula (4.1) reduces to

$$2^{2\rho+1/2}z^{2\rho}e^{-z^2/2}H^{0,\lambda:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A,C:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: \\ [(c):\Psi',\ldots,\Psi^{(n)}]:\end{matrix}\right. \tag{5.9}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}x_1z^{2\alpha_1},\ldots,x_nz^{2\alpha_n}\right)\prod_{i=1}^{r}P_{n_i}^{(\xi_i,\eta_i)}(1-2y_iz^{2\beta_i})$$

$$=\sum_{s=0}^{\infty}H_s(Z)\frac{2^s}{s!}\sum_{k_1=0}^{n_1}\cdots\sum_{k_r=0}^{n_r}\prod_{i=1}^{r}\frac{(-n_i)_{k_i}}{k_i!}\frac{y_i^{k_i}}{4^{\beta_i k_i}}\binom{n_i+\xi_i}{n_i}\frac{(1+\xi_i+\eta_i+n_i)}{(1+\xi_i)_{k_i}}$$

$$H^{0,\lambda+1:(\mu',\nu');\ldots;(\mu^{(n)},\nu^{(n)})}_{A+1,C+1:(B',D');\ldots;(B^{(n)},D^{(n)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]: & [-2\rho-2\beta_1k_1-\cdots-2\beta_rk_r:2\alpha_1,\ldots,2\alpha_n]; \\ [(c):\Psi',\ldots,\Psi^{(n)}]: & \left[\frac{s}{2}-\rho-\beta_1k_1-\cdots-\beta_rk_r:\alpha_1,\ldots,\alpha_n\right];\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]; \\ [(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\frac{x_1}{4^{\alpha_1}},\ldots,\frac{x_n}{4^{\alpha_n}}\right),$$

provided that all conditions of (2.1) hold true.

Similarly specializing the parameters of *H*-function, we can derive the similar results involving different special functions which may be useful in mathematical analysis and physical problems.

REFERENCES

[1] Bhonsle, B.R., Heat Conduction and Hermite Polynomials, *Proc. Nat. Acad. Sci., India,* **36** (1966), 359-360.

[2] Chandel, R.C.S. and Bhargava, S.K., A problem on the cooling of a heated cylinder, Jour. *Maulana Azad College Tech.*,**15** (1982), 99-103

[3] Chandel, R.C.S. and Yadava, H.C., Heat conduction and multiple hypergeometric function of Srivastava and Daoust, *Indian J. Pure. Appl. Math.* **15(4),** (1984), 371-376.

[4] Chandel, R.C.S. and Gupta, A.K., Head Conduction and *H*-function of several variables, *Jour. Maulana Azad College, Tech.,* **17** (1984), 85-92.

[5] Chandel, R.C.S. and Gupta, A.K., Use of multivariable *H*-function of Srivastava and Panda in cooling of a heated cylinder, *Pure Appl. Math. Sci.,* **25** (1987), 43-48.

[6] Chandel, R.C.S. and Tiwari, A., Multiple hypergeometric function of Srivastava and Daoust and its applications in two boundary value problems, *Jñānābha* **23** (1993), 97-103.

[7] Chandel, R.C.S. and Sengar, S., On two boundary value problems, *Jñānābha* **31/32** (2002), 89-103.

[8] Chandel, R.C.S. and Sengar, S., A problem of heat conduction in a rod under the Robin condition, *Jñānābha,* **33** (2003), 131-138.

[9] Chandel, R.C.S. and Singh, Y.K., Two boundary value problems, *Indian J. of Theoretical Physics,* **53**(4) (2005), 339-350.

[10] Chandel R.C.S. and Tiwari, K.P., A generalization of multivariable polynomials, *Jñānābha,* **38** (2008), 153-160.

[11] Chandel, R.C.S. and Sengar Shailja, Multivariable generalized polynomials defined through their generating function, *Jñānābha*, **40** (2010), 105-112.

[12] Chandel, R.C.S. and Sharma, S., A multivariable analogue of a class of polynomials, *Jñānābha*, **45** (2015), 95-102.

[13] Chandel, R.C.S. and Kumar Hemant, Contour Integral representation of two variable generalized hypergeometric functions of Srivastava and Daoust with their applications to initial value problems of arbitrary order, *Jñānābha*, **50** (Jun 2020), 232-242.

[14] Chaurasia, V.B.L. and Patni, R., A heat conduction problem and certain product of the multivariable *H*-function with two general classes of polynomials, *Jñānābha*, **27** (1997), 1-9.

[15] Chaurasia, V.B.L. and Gupta, N., A solution of partial differential equation of heat conduction in a rod under Robin condition, *Jñānābha*, **27** (1997), 31-45.

[16] Churchill, R.V., *Operational Mathematics,* Mc Graw-Hill, New York, 1958.

[17] Erdelyi, A. et al., *Tables of Integral Transforms,* **2**, McGrawhill Book Company, Inc., 1954.

[18] Kumar Hemant and Rai Suryakant, Multiple Fractional Diffusions via multivariable *H*-function, **50** (June 2020), 253-264.

[19] Kampe de Feriet, Heat conduction and Hermite polynomials, *Bull. Calcutta Math. Soc.,* **The Golden Jubilee Commemoration Vol.** (1958-59), 193-204.

[20] Lebdev, N.N., *Special Function and Their Applications*, Prentice Hall Inc. Engle Wood, Cliffs, N.J., 1965.

[21] Olsson, P.O.M., A hypergeometric function of two variables of importance in perturbation theory, I, II, *Ark. Fys.* **29** (1964), 459-465, **30** (1965), 187-191.

[22] Rainville, E.D., *Special Functions,* Macmillan Company, New York, 1965; Reprinted by Chelsea Pub. Co., Bronx., New York. 1971.

[23] Singh, F., Use of generalized hypergeometric function in cooling of a heated cylinder, *Math. Education (Siwan)* **3A** (1969), 37-40.

[24] Singh, F., Expansion formulae for Kampe de Feriet and radial wave functions and heat conduction, *Def. Sci. Jour.*, 21 (1971), 265-272.

[25] Srivastava, H.M., A contour integral involving Fox's *H*-function, *Indian J. Math.,* **14** (1972), 1-6.

[26] Srivastava, H.M. and Daoust, M.C., Certain generalized meumann expansions associated with the Kampe de Feriet function, *Nederl. Akad. Wetensch, Proc. Ser.* A **72**, *Indag. Math.,* **31** (1969), 449.

[27] Srivastava, H.M. and Daoust M.C., On enterian integral associated with Kampe de Feriet's function *Publ. Insdt. Math. (Beograd) (N.S.),* **9 (23)** (1969), 199-202.

[28] Srivastava, H.M. and Daoust, M.C., A note on the convergence of Kampe de Feriet's double hypergeometric series, *Math. Nachr.,* **53** (1972), 151-159.

[29] Srivastava, H.M. and Karlsson, P.W., *Multiple Gaussian* Hypergeometric Series, John Wiley and Sons, New York, 1985.

[30] Srivastava, H.M. and Panda, R., Some expansion theorems and generating relations for the *H*-function of several complex variables, *Comment. Math. Univ. St. Paul.* **24** fasc. **2** (1975). 119-137.

[31] Srivastava, H.M. and Panda, R., Expansion theorems for the *H*-function of several complex variables, *J. Reine Angew. Math.* **288** (1976), 129-145.

[32] Srivastava, H.M. and Panda, R., Some bilateral generating functions for a class of generalized hypergeometric polynomials, *J. Reine Angew Math.*, **283/284** (1976), 265-274.

[33] Srivastava, H.M., Gupta, K.C. and Goyal, S.P., The *H-Functions of One and Two Variables with Applications,* South Asian Publishers, New Delhi, 1982.

[34] Srivastava, H.M. and Singh, N.P., The integration of certain products of multivariable *H*-function with general class of polynomials, *Rend. Circ. Mat. Palermo,* (2) **32** (1983), 157-187.

[35] Szego, G., Orthogonal polynomials, *Amer. Math. Soc. Colloq. Publ.* Vol. **23**, Fourth Edition, *Amer. Math. Soc.*, Providence, Rohde Island, 1975.