J. Nat. Acad. Math. ISSN 0970-5228
Vol. 35 (2021), pp.42-51

Role of Pressure Gradient and Porosity Parameter in Enhancing Mucus Transport in Lung Airways

By

V.S. Verma, Janta Raut and Vikash Rana

Department of Mathematics and Statistics, Deen Dayal Upadhyaya, Gorakhpur University, Gorakhpur

Email: drvsverma01@gmail.com

Abstract

In this paper, a two-layer unsteady state planar mathematical model is proposed to study mucus transport in human lung airways by considering mucus as a viscoelastic fluid and serous layer as a Newtonian fluid by taking into account the effect of porosity parameter due to immotile cilia forming porous matrix in the serous sub-layer in contact with the epithelium. The effect of shear stress induced by air-motion due to forced expiration and acceleration due to gravitational force are also considered in the model. It is observed that mucus transport rate increases as the pressure gradient, the shear stress and porosity parameter increase, but it decreases as its viscosity increases. It is also observed that mucus transport rate decreases as the viscosity of serous layer fluid increases, but any increase in mucus viscosity at its higher values does not seem to affect the mucus transport. It is also shown that mucus transport rate decreases as its modulus of elasticity or its density increases. It is also observed that the mucus transport rate decreases as the thickness of serous layer fluid or transport duration increases.

Keywords: human lung airways, immotile cilia, mucus transport, viscosity, porosity.
Mathematics Subject Classification: 76A05, 76A10, 76D05, 76D10, 92B05

1. Introduction

The normal mechanics of mucus transport in the human lung airways is constantly swept from upper to lower airways to remove mucus. Several investigators have studied the mechanics of mucus transport theoretically and several experiments have been conducted to study how mucus flows in human lung airways in healthy and unhealthy states. (Agarwal and Verma [1], Blake [2], Hanliang and Kanso [3], King et al. [4], Verma and Tripathee [5], Verma and Rana [6], Verma [7, 8], etc.). Some primitive mathematical models were developed based on the motion of motile cilia by considering the effects of flows of serous layer fluid and mucus on cilia. Some investigators have incorporated the effects of visco-elasticity, flow duration, the thicknesses and viscosities of serous layer fluid and mucus on mucus transport mechanics.

Agarwal and Verma [1] have presented a two-layer planar unsteady state model for mucus transport due to coughing. They have used Dirac-delta function for the pressure gradient generated due to coughing and analyzed the results graphically. Blake [2] discussed the role of acceleration due to gravity and airflow on mucus transport in two layers Newtonian fluid model. Hanliang and Kanso [3] have studied mucociliary transport in healthy and diseased environments and shown that a healthy mucus layer enhances the performance of mucociliary system. In contrast, in the diseased environments, mucus hinders transport and stiffer mucus leads to a substantial decrease in transport efficiency. King et al. [4]

Received: April 17, 2021; Accepted: September 09, 2021

have developed a mathematical model to study the effect of mucus visco-elasticity on mucus transport. Verma and Tripathee [5] have studied mucus transport in human lung airways by taking the effects of mucus visco-elasticity, air-motion, cilia beating and porosity parameter in steady state. They have presented and analysed a two-layer circular steady state mathematical model. Verma and Rana [6] have studied mucus transport in the human lung airways by presenting two-layer fluid model taking mucus as a visco-elastic fluid. Verma [7,8] has presented planar as well as symmetrical mathematical models to estimate the rate of mucus transport in human lung airways by taking mucus as a visco-elastic fluid. The Laplace transform technique has been used in solving the models. It has been found that mucus transport rate increases when the shear stress caused by air-motion and the pressure gradient increase. It has been also found that an increase in the elastic modulus of mucus results a decrease in its transport rate.

In view of above, in this paper, we present a two-layer unsteady state planar mathematical model to study the transport of serous layer fluid and mucus in the human lung airways by taking the effects of the pressure gradient, the shear stress induced by air-motion, acceleration due to gravitational force and porosity parameter due to certain immotile cilia forming porous matrix by considering serous fluid as Newtonian and mucus as visco-elastic fluid.

2. Mathematical Model

The physical situation of transport of serous fluid and mucus is shown in Fig.1. The model consists of two fluid layers; namely serous fluid layer and mucus layer. The serous fluid is considered as Newtonian fluid while mucus is considered as visco-elastic fluid. Again, the serous fluid layer is divided into two sub-layers; one in touch with the epithelium and other in touch with mucus layer. It is assumed that a porous matrix is formed by certain immotile cilia in serous sub-layer in touch with the epithelium. No net flow is assumed in the serous sub-layer in touch with the epithelium. The effects the shear stress induced by air-motion, the pressure gradient and acceleration due to gravitational force are also considered in the model.

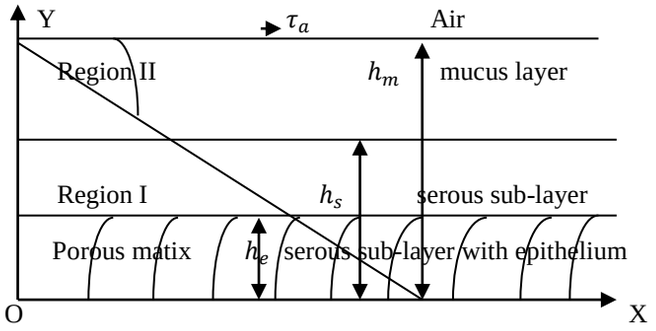

Figure 1. A planar two- layer fluid model for mucus transport

The equations describing the transport of mucus and serous layer fluid under the unsteady state condition and low Reynolds number flow approximations are taken as follows:

Region-I: Serous layer ($h_e \leq y \leq h_s$):

$$\rho_s \frac{\partial u_s}{\partial t} = -\left(\frac{\partial p}{\partial x} - \rho_s g \cos\theta\right) + \mu_s \frac{\partial^2 u_s}{\partial y^2} \tag{1}$$

Region-II: Mucus layer ($h_s \leq y \leq h_m$):

$$\rho_m \frac{\partial u_m}{\partial t} = -\left(\frac{\partial p}{\partial x} - \rho_m g \cos\theta\right) + \frac{\partial \tau_m}{\partial y} \tag{2}$$

$$\tau_m + \lambda \frac{\partial \tau_m}{\partial t} = \mu_m \frac{\partial u_m}{\partial y} \tag{3}$$

where t is the time, p represents constant pressure across the layers; u_s and u_m are the velocity components of the serous fluid and mucus in x-direction respectively. ρ_s, ρ_m are densities and μ_s, μ_m are viscosities of serous fluid and mucus respectively; τ_m is the shear stress in the mucus layer; $\lambda(=\frac{\mu_m}{G})$ is the relaxation time; G is the modulus of elasticity, g is the acceleration due to gravitational force and θ is the angle made by the airway with the vertical. Here, h_e denotes the average thickness measured from the epithelium's surface to the tips of cilia forming porous matrix, h_s denotes the thickness measured from the epithelium's surface to the surface between the serous sub-layer and mucus; and h_m denotes the thickness measured from the epithelium's surface to the mucus-air interface. Equation (3) provides a relationship between the shear stress and rate of strain for a visco-elastic fluid. The following are chosen as initial, boundary and matching conditions for the system of equations (1)-(3).

Initial Conditions

$$u_s = 0,\ u_m = 0, \tau_m = 0, \frac{\partial \tau_m}{\partial t} = 0, \frac{\partial u_m}{\partial y} = 0 \qquad \text{at } t = 0 \tag{4}$$

Boundary Conditions

$$u_s = \beta \frac{\partial u_s}{\partial y}, \qquad y = h_e, t > 0 \tag{5}$$

$$\tau_m = \tau_a, \qquad y = h_m, t > 0 \tag{6}$$

where β is the porosity parameter due to porous matrix formed by immotile cilia. Condition (5) incorporates the effect of immotile cilia, which happens in some diseased state where cilia beating ceases. Condition (6) incorporates the shear stress caused by the air-motion.

Matching Conditions

$$u_s = u_m = U_1, \qquad y = h_s \tag{7}$$

$$\mu_s \frac{\partial u_s}{\partial y} = \tau_m, \qquad y = h_s \tag{8}$$

where U_1 denotes the velocity at the mucus-serous interface which is to be determined. The conditions (7) and (8) imply that velocities and shear stresses that are continuous at the interface of mucus-serous layer.

3. Analytical Solution

Solving (1)-(3) and applying initial, boundary and matching conditions (4)-(8) , we get

$$\bar{u}_s = \bar{U}_1 \left[\frac{\sinh k_s(y-h_e) + \beta k_s \cosh k_s(y-h_e)}{\sinh k_s(h_s-h_e) + \beta k_s \cosh k_s(h_s-h_e)}\right] - \frac{\bar{\phi}_s}{S\rho_s}\left[\frac{\sinh k_s(h_s-h_e)+\sinh k_s(y-h_s)-\sinh k_s(y-h_e)+\beta k_s\{\cosh k_s(h_s-h_e)-\cosh k_s(y-h_e)\}}{\sinh k_s(h_s-h_e)+\beta k_s \cosh k_s(h_s-h_e)}\right] \tag{9}$$

$$\bar{u}_m = \bar{U}_1 \frac{\cosh k_m(y-h_m)}{\cosh k_m(h_m-h_s)} + \frac{k_m \bar{\tau}_a}{S\rho_m} \frac{\sinh k_m(y-h_s)}{\cosh k_m(h_m-h_s)} - \frac{\bar{\phi}_m}{S\rho_m}\left[\frac{\cosh k_m(h_m-h_s) - \cosh k_m(y-h_m)}{\cosh k_m(h_m-h_s)}\right] \tag{10}$$

where $\bar{U}_1$ is given by

$$\bar{U}_1\left[\frac{\mu_s k_s \coth k_s(h_s-h_e)+\beta\mu_s {k_s}^2}{1+\beta k_s \coth k_s(h_s-h_e)} + k_m\lambda(S)\tanh k_m(h_m-h_s)\right] = \bar{\tau}_a \operatorname{sech} k_m(h_m-h_s)$$

$$-\frac{\bar{\phi}_m}{k_m}\tanh k_m(h_m-h_s) - \frac{\bar{\phi}_s}{k_s}\left[\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\beta k_s}{1+\beta k_s \coth k_s(h_s-h_e)}\right] \tag{11}$$

$${k_s}^2 = \frac{S\rho_s}{\mu_s}, {k_m}^2 = \frac{\rho_m}{G}S(S+\alpha), \lambda(S) = \frac{\mu_m}{1+\lambda S}, \alpha = \frac{G}{\mu_m}, \bar{\phi}_s = \frac{\partial p}{\partial x} - \rho_s g\cos\theta, \bar{\phi}_m = \frac{\partial p}{\partial x} - \rho_m g\cos\theta$$

where S is Laplace transform variable and bar $(-)$ denotes Laplace transform of the corresponding functions.

The volumetric transport rates in the two layers (serous and mucus) are respectively defined as follows:

$$\bar{Q}_s = \int_{h_e}^{h_s} \bar{u}_s\, dy \quad \text{and} \quad \bar{Q}_m = \int_{h_s}^{h_m} \bar{u}_m\, dy$$

which after using equations (9) and (10) computed and are given below:

$$\bar{Q}_s = \frac{\bar{U}_1}{k_s}\left[\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\beta k_s}{1+\beta k_s \coth k_s(h_s-h_e)}\right] - \frac{\bar{\phi}_s}{S\rho_s}\left[(h_s-h_e) - \frac{2}{k_s}\left\{\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\frac{1}{2}\beta k_s}{1+\beta k_s \coth k_s(h_s-h_e)}\right\}\right] \tag{12}$$

$$\bar{Q}_m = \frac{\bar{U}_1}{k_m}\tanh k_m(h_m-h_s) + \frac{\bar{\tau}_a}{S\rho_m}[1-\operatorname{sech} k_m(h_m-h_s)] - \frac{\bar{\phi}_m}{S\rho_m}\left[(h_m-h_s) - \frac{1}{k_m}\tanh k_m(h_m-h_s)\right] \tag{13}$$

Here, $\bar{Q}_s$ and $\bar{Q}_m$ are constants as fluid continuity equation is considerable. Hence, from equations (12) and (13), it can be noted that $\left(-\frac{\partial p}{\partial x}\right)$ is also constant. So, it can be replaced by the pressure drop over the length 'L' of the cilia forming porous matrix zone. Thus, in this case, the expressions for transport rates brcome:

$$\bar{Q}_s = \frac{\bar{U}_1}{k_s}\left[\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\beta k_s}{1+\beta k_s \coth k_s(h_s-h_e)}\right] + \frac{\bar{\phi}_{so}}{S\rho_s}\left[(h_s-h_e) - \frac{2}{k_s}\left\{\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\frac{1}{2}\beta k_s}{1+\beta k_s \coth k_s(h_s-h_e)}\right\}\right] \tag{14}$$

$$\bar{Q}_m = \frac{\bar{U}_1}{k_m}\tanh k_m(h_m-h_s) + \frac{\bar{\tau}_a}{S\rho_m}[1-\operatorname{sech} k_m(h_m-h_s)] + \frac{\bar{\phi}_{mo}}{S\rho_m}\left[(h_m-h_s) - \frac{1}{k_m}\tanh k_m(h_m-h_s)\right] \tag{15}$$

where $\bar{\phi}_{so} = \frac{\Delta p}{L} + \rho_s g\cos\theta$, $\bar{\phi}_{mo} = \frac{\Delta p}{L} + \rho_m g\cos\theta$, $\Delta p = p_o - p_L$; $p = p_o$ at $x = 0$, $p = p_L$ at $x = L$ and U_1 is given by

$$\bar{U}_1\left[\frac{\mu_s k_s \coth k_s(h_s-h_e)+\beta\mu_s {k_s}^2}{1+\beta k_s \coth k_s(h_s-h_e)} + k_m\lambda(S)\tanh k_m(h_m-h_s)\right] =$$

$$\bar{\tau}_a \operatorname{sech} k_m(h_m-h_s) + \frac{\bar{\phi}_{mo}}{k_m}\tanh k_m(h_m-h_s)$$

$$+\frac{\bar{\phi}_{so}}{k_s}\left[\frac{\tanh\frac{k_s(h_s-h_e)}{2}+\beta k_s}{1+\beta k_s\coth k_s(h_s-h_e)}\right] \tag{16}$$

Here, the acceleration due to gravitational force and the pressure gradient have similar effects.

Since it is not easy to find the exact inverse transform of $\bar{Q}_s$ and $\bar{Q}_m$ given by (14) and (15), therefore, the expressions can be approximated by using reasonable assumptions (i.e. small thickness of serous layer fluid and large mucus thickness) i.e. $k_s(h_s-h_e)\ll 1$ and $k_m(h_m-h_s)\gg 1$. In such case, equations (14) and (15) are approximated as follows:

$$\bar{Q}_s=\frac{\phi_{mo}}{2\mu_s}\left[\frac{1}{S^{3/2}(S+\alpha)^{1/2}}-\frac{(G\rho_m)^{\frac{1}{2}}}{\mu_s}\frac{(h_s-h_e)}{S(S+\alpha)}-\beta\frac{\rho_s}{\mu_s}\frac{(h_s-h_e)}{S^{1/2}(S+\alpha)^{1/2}}\right]\left(\frac{G}{\rho_m}\right)^{\frac{1}{2}}(h_s-h_e)(h_s-h_e+2\beta)+$$
$$\frac{\phi_{so}}{4\mu_s}\left[\frac{1}{S}-\frac{(G\rho_m)^{\frac{1}{2}}}{\mu_s}\frac{(h_s-h_e)}{S^{1/2}(S+\alpha)^{1/2}}-\beta\frac{\rho_s}{\mu_s}(h_s-h_e)\right]\frac{(h_s-h_e)^2(h_s-h_e+2\beta)^2}{(h_s-h_e+\beta)} \tag{17}$$

$$\bar{Q}_m=$$
$$\frac{1}{\rho_m}\left[\{\tau_a+\phi_{mo}(h_m-h_s)\}\frac{1}{S^2}-\frac{\phi_{mo}\left(G/\rho_m\right)^{\frac{1}{2}}}{S^{5/2}(S+\alpha)^{1/2}}\right]+$$
$$\frac{\phi_{mo}}{\mu_s}\left[\frac{1}{S^2(S+\alpha)}-\frac{(G\rho_m)^{\frac{1}{2}}}{\mu_s}\frac{(h_s-h_e)}{S^{3/2}(S+\alpha)^{3/2}}-\beta\frac{\rho_s}{\mu_s}\frac{(h_s-h_e)}{S(S+\alpha)}\right]\left(\frac{G}{\rho_m}\right)(h_s-h_e+\beta)+$$
$$\frac{\phi_{so}}{2\mu_s}\left[\frac{1}{S^{3/2}(S+\alpha)^{1/2}}-\frac{(G\rho_m)^{\frac{1}{2}}}{\mu_s}\frac{(h_s-h_e)}{S(S+\alpha)}-\frac{\rho_s\beta}{\mu_s}\frac{(h_s-h_e)}{S^{1/2}(S+\alpha)^{1/2}}\right]\left(\frac{G}{\rho_m}\right)^{\frac{1}{2}}(h_s-h_e)(h_s-h_e+2\beta) \tag{18}$$

Now, taking inverse Laplace transform of (17) and (18), the transport rates can be found as follows:

$$Q_s=\frac{\phi_{mo}}{2\mu_s}\left[\left(\frac{G}{\rho_m}\right)^{\frac{1}{2}}te_2\left\{I_0\left(\frac{1}{2}\alpha t\right)+I_1\left(\frac{1}{2}\alpha t\right)\right\}-\left(\frac{\mu_m}{\mu_s}\right)(h_s-h_e+\beta)(1-e_1)\right](h_s-h_e)(h_s-h_e+2\beta)$$
$$+\frac{\phi_{so}}{4\mu_s}\left[1-\frac{(G\rho_m)^{\frac{1}{2}}}{\mu_s}(h_s-h_e+\beta)e_2I_0\left(\frac{1}{2}\alpha t\right)\right.$$
$$\left.-\beta\frac{\rho_s}{\mu_s}(h_s-h_e)\right]\frac{(h_s-h_e)^2(h_s-h_e+2\beta)^2}{(h_s-h_e+\beta)} \tag{19}$$

$$Q_m=\left\{\frac{\phi_{so}}{2\mu_s}(h_s-h_e)(h_s-h_e+2\beta)\right\}\left[\left(\frac{G}{\rho_m}\right)^{\frac{1}{2}}te_2\left\{I_0\left(\frac{1}{2}\alpha t\right)+I_1\left(\frac{1}{2}\alpha t\right)\right\}-\left(\frac{\mu_m}{\mu_s}\right)(h_s-h_e+\beta)(1-e_1)\right]$$
$$+\frac{t}{\rho_m}\left[\tau_a+\phi_{mo}\left\{(h_m-h_s)+\left(\frac{\mu_m}{\mu_s}\right)(h_s-h_e+\beta)+\frac{(\mu_m e_2)}{2(G\rho_m)^{\frac{1}{2}}}(8+\alpha t)\right\}\right]$$
$$-\frac{\phi_{mo}}{\rho_m}\left[\frac{8{\mu_m}^2}{G}\frac{1}{(G\rho_m)^{\frac{1}{2}}}(1-e_2)\right.$$
$$\left.+\left\{\frac{2}{\mu_s}(h_s-h_e+\beta)(G\rho_m)^{\frac{1}{2}}te_2I_1\left(\frac{1}{2}\alpha t\right)+\left(\frac{\mu_m}{G}\right)(1-e_1)\right\}\left(\frac{\mu_m}{\mu_s}\right)(h_s-h_e+\beta)\right]$$

$$-\beta\left[\left\{U_0+\frac{\phi_{so}}{2\mu_s}(h_s-h_e)(h_s-h_e+\beta)\right\}\left(\frac{G}{\rho_m}\right)^{\frac{1}{2}}e_2 I_0\left(\tfrac{1}{2}\alpha t\right)+\frac{\phi_{mo}}{\rho_m}\left(\frac{\mu_m}{\mu_s}\right)(h_s-h_e+\beta)(1-e_1)\right]\left(\frac{\rho_s}{\mu_s}\right)(h_s-h_e) \quad (20)$$

where $e_1=\exp(-\alpha t)$, $e_2=\exp\left(-\frac{1}{2}\alpha t\right)$ and I_0, I_1 are the modified Bessel functions of the first kind of order zero and unity respectively [6].

4. Results and Discussion

To see the effects of distinct parameters on mucus transport rate quantitatively, the expression for Q_m given by (20) by using the non-dimensional parameters: ${\mu_s}^*=\frac{\mu_s}{\mu_o}, {\rho_s}^*=\frac{h_m\rho_s U_o}{\mu_o}, {\phi_{so}}^*=\frac{{h_m}^2\phi_s}{\mu_o U_o}, {\mu_m}^*=\frac{\mu_m}{\mu_o}, {\rho_m}^*=\frac{h_m\rho_m U_o}{\mu_o},,{\phi_{mo}}^*=\frac{{h_m}^2\phi_m}{\mu_o U_o}, G^*=\frac{h_m G}{\mu_o U_o}, {h_e}^*=\frac{h_e}{h_m}, {h_s}^*=\frac{h_s}{h_m}, {\tau_a}^*=\frac{h_m\tau_a}{\mu_o U_o}$, $\alpha^*=\frac{G^*}{{\mu_m}^*}, \beta^*=\frac{\beta}{h_m}, t^*=\frac{U_o t}{h_m}$ and ${Q_m}^*=\frac{Q_m}{h_m U_o}$ (μ_o being the viscosity of serous fluid in contact with epithelium) can be written in non-dimensional form as follows:

$$Q_m{}^*=\left\{\frac{{\phi_{so}}^*}{2{\mu_s}^*}({h_s}^*-{h_e}^*)H_2^*\right\}\left[\left(\frac{G^*}{{\rho_m}^*}\right)^{\frac{1}{2}}t^*{e_2}^*\left\{I_0\left(\frac{1}{2}\alpha^*t^*\right)+I_1\left(\frac{1}{2}\alpha^*t^*\right)\right\}-\left(\frac{{\mu_m}^*}{{\mu_s}^*}\right)H_1^*(1-{e_1}^*)\right]$$

$$+\frac{t^*}{{\rho_m}^*}\left[{\tau_a}^*+{\phi_{mo}}^*\left\{(1-{h_s}^*)+\left(\frac{{\mu_m}^*}{{\mu_s}^*}\right)H_1^*+\frac{({\mu_m}^*{e_2}^*)}{2(G^*{\rho_m}^*)^{\frac{1}{2}}}(8+\alpha^*t^*)\right\}\right]$$

$$-\frac{{\phi_{mo}}^*}{{\rho_m}^*}\left[\frac{8{{\mu_m}^*}^2}{G^*}\frac{(1-{e_2}^*)}{(G^*{\rho_m}^*)^{\frac{1}{2}}}+\left(\frac{{\mu_m}^*}{{\mu_s}^*}\right)\left\{\frac{2}{{\mu_s}^*}H_1^*(G^*{\rho_m}^*)^{\frac{1}{2}}t^*{e_2}^*I_1\left(\frac{1}{2}\alpha^*t^*\right)+\left(\frac{{\mu_m}^*}{G^*}\right)(1-{e_1}^*)\right\}H_1^*\right]$$

$$-\beta^*\left[\left\{1+\frac{{\phi_{so}}^*}{2{\mu_s}^*}({h_s}^*-{h_e}^*)H_1^*\right\}\left(\frac{G^*}{{\rho_m}^*}\right)^{\frac{1}{2}}{e_2}^*I_0\left(\frac{1}{2}\alpha^*t^*\right)+\frac{{\phi_{mo}}^*}{{\rho_m}^*}\left(\frac{{\mu_m}^*}{{\mu_s}^*}\right)H_1^*(1-{e_1}^*)\right]\left(\frac{{\rho_s}^*}{{\mu_s}^*}\right)({h_s}^*-{h_e}^*) \quad (21)$$

where H_1^* and H_2^* are respectively given by $H_1^*=h_s-h_e+\beta$ and $H_2^*=h_s-h_e+2\beta$.

Various graphs are plotted in Figures (2)-(8), by taking following set of parameters which have been calculated by using typical values of various characteristics related to human lung airways [3,5].

${\mu_s}^*=1-10$, ${\rho_s}^*=1.0$, ${\phi_{so}}^*=1.0$, ${\mu_m}^*=100-1000$, ${\rho_m}^*=0.2-0.5$, ${\phi_{mo}}^*=2-25$, $G^*=10-100$, ${h_e}^*=0.1, {h_s}^*=0.2-0.6, \beta^*=0.02-0.10, {\tau_a}^*=1-10, t^*=0.10-0.16$.

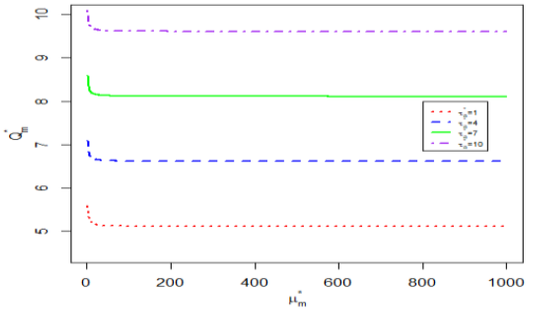

Figure 2. Variation of ${Q_m}^*$ with ${\mu_m}^*$ for distinct values of ${\tau_a}^*$

Fig.2 shows the variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $\tau_a{}^*$ and fixed values of $h_s{}^* = 0.2, h_e{}^* = 0.1, \beta^* = 0.02,\ \mu_s{}^* = 1, \rho_m{}^* = 0.2, G^* = 20,\ \phi_{mo}{}^* = 25, \phi_{so}{}^* = 1.0\ \rho_s{}^* = 1.0,, t^* = 1 \times 10^{-1}$. This figure demonstrates that the mucus transport rate decreases as its viscosity increases , but it increases when the shear stress induced by air-flow at mucus-air interface increases. These observations are in line with analytical results of [3,4].

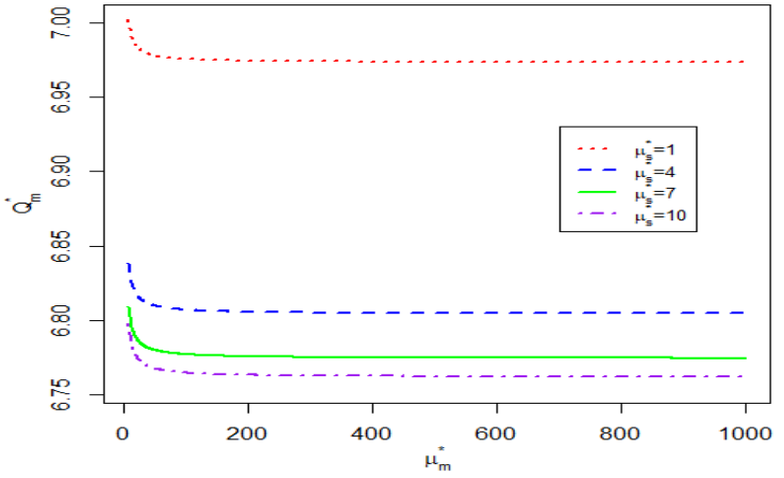

Figure 3. Variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $\mu_s{}^*$

Fig. 3 shows the variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $\mu_s{}^*$ and fixed values of $\tau_a{}^* = 9, h_s{}^* = 0.2, h_e{}^* = 0.1, \beta^* = 0.02, G^* = 10,\ \phi_{mo}{}^* = 10, \phi_{so}{}^* = 1.0, \rho_s{}^* = 1.0, \rho_m{}^* = 0.2, t^* = 1 \times 10^{-1}$. This figure illustrates that mucus transport rate decreases when the viscosity of the serous fluid or that of mucus increases. These observations are in line with analytical results of [3].

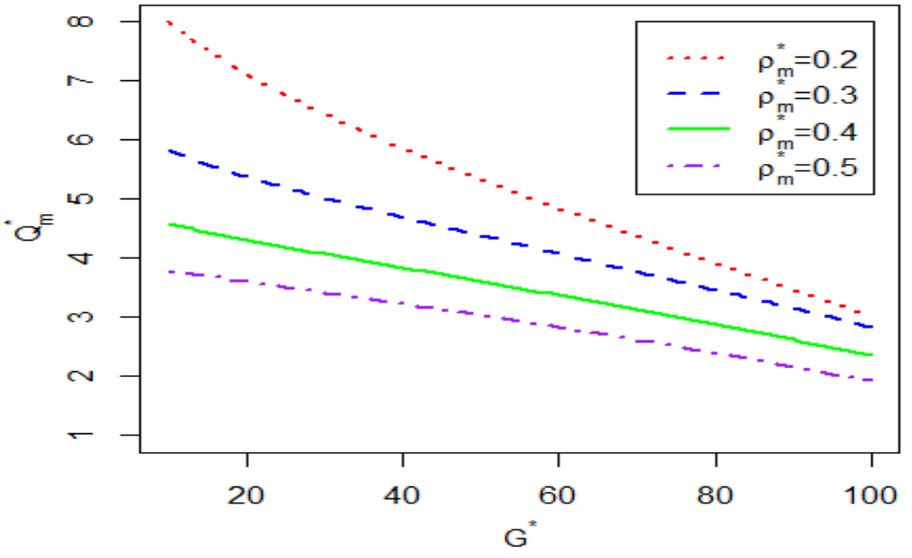

Figure 4. Variation of $Q_m{}^*$ with G^* for distinct values of $\rho_m{}^*$

Fig.4 shows the variation of $Q_m{}^*$ with G^* for distinct values of $\rho_m{}^*$ and fixed values of $\tau_a{}^* = 5, h_s{}^* = 0.2, h_e{}^* = 0.1, \beta^* = 0.02, \phi_{mo}{}^* = 20, \phi_{so}{}^* = 1.0, \rho_s{}^* = 1.0\ , \mu_s{}^* = 1, t^* = 1 \times 10^{-1}$.This figure illustrates that mucus transport rate decreases as the modulus of elasticity increases. The figure also illustrates that mucus transport rate decreases as its density increases for constant elastic modulus. These observations are in line with analytical results of [3,7].

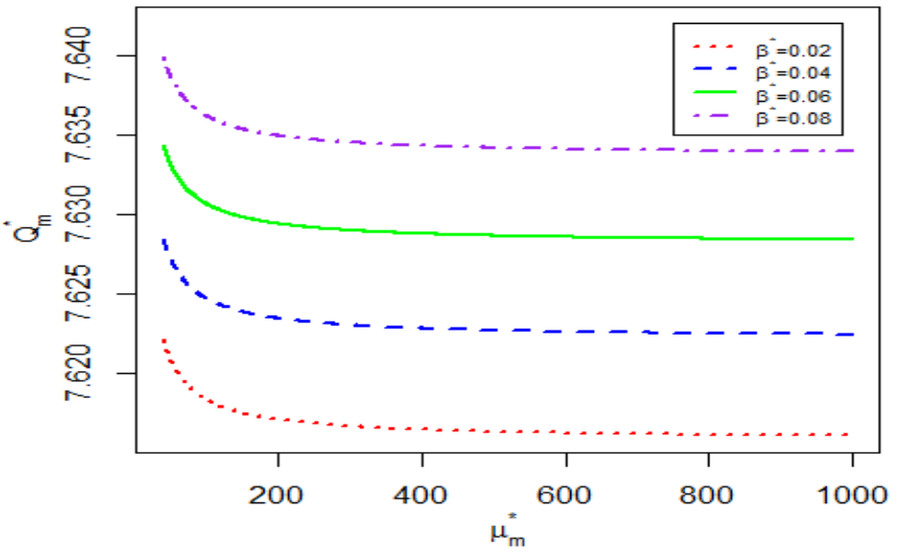

Figure 5. Variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of β^*

Figure 5 shows the variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of β^* and fixed values of $\tau_a{}^* = 7, h_s{}^* = 0.2, h_e{}^* = 0.1, G^* = 10, \phi_{mo}{}^* = 15, \phi_{so}{}^* = 1.0, \rho_s{}^* = 1.0, \rho_m{}^* = 0.2, \mu_s{}^* = 5, t^* = 1 \times 10^{-1}$. From this figure, it is clear that mucus transport rate decreases as the mucus viscosity increases, but increase in porosity parameter results an increase in mucus transport rate. These observations are in line with analytical results of [5,7].

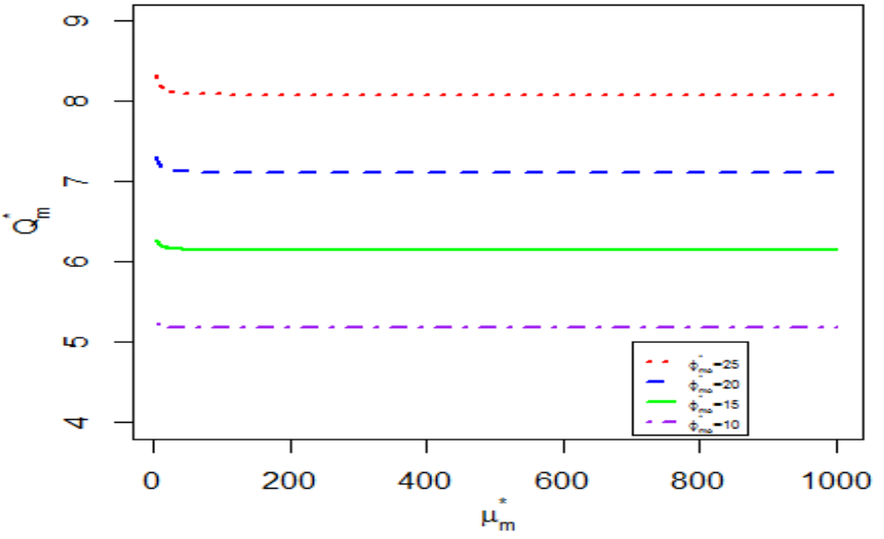

Figure 6. Variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $\phi_{mo}{}^*$

Fig.6 shows the variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $\phi_{mo}{}^*$ and fixed values of $\tau_a{}^* = 5, h_s{}^* = 0.2, h_e{}^* = 0.1, \beta^* = 0.02, \mu_s{}^* = 1.0\,, \phi_{so}{}^* = 1.0,\ \rho_m{}^* = 0.2, \rho_s{}^* = 1.0, t^* = 1 \times 10^{-1}$. This figure shows that mucus transport rate decreases when its viscosity increases. It can also seen from this figure that an increase in pressure gradient enhances the mucus transport rate. These observations are in line with analytical results of [3].

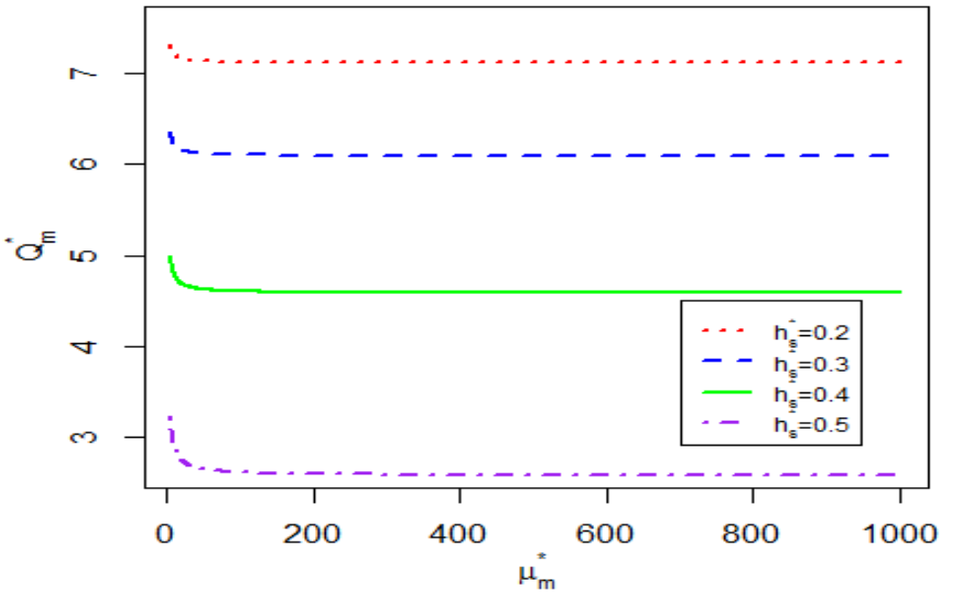

Figure 7. Variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $h_s{}^*$

Fig.7 shows the variation of $Q_m{}^*$ with $\mu_m{}^*$ for distinct values of $h_s{}^*$ and fixed values of $\tau_a{}^* = 5$, $h_e{}^* = 0.1, \beta^* = 0.02, \mu_s{}^* = 1.0, \phi_{mo}{}^* = 20, \phi_{so}{}^* = 1.0,\ \rho_m{}^* = 0.2, \rho_s{}^* = 1.0, G^* = 20, t^* = 1 \times 10^{-1}$. This figure shows that mucus transport rate decreases when mucus viscosity or thickness of serous layer fluid increases, but increase in viscosity of mucus for its at higher values does not have any significant effect on its transport rate, which is an indication that the mucus behaves as an elastic slab at the higher values of its viscosity . These observations are in line with analytical results of [5,8].

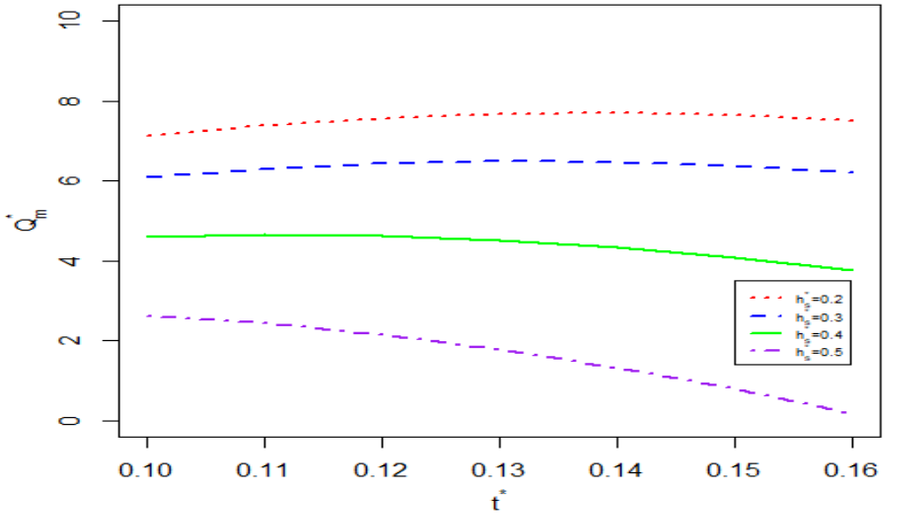

Figure 8. Variation of $Q_m{}^*$ with t^* for distinct values of $h_s{}^*$

Fig.8 shows the variation of $Q_m{}^*$ with t^* for distinct values of $h_s{}^*$ and fixed values of $\tau_a{}^* = 5$, $h_e{}^* = 0.1, \beta^* = 0.02, \mu_m{}^* = 100, \mu_s{}^* = 1.0, \phi_{mo}{}^* = 20, \phi_{so}{}^* = 1.0,\ \rho_m{}^* = 0.2, \rho_s{}^* = 1.0, G^* = 20$. This figure shows that mucus transport rate decreases with increase in transport duration. Here, it is also seen that the mucus transport rate decreases with an increase in thickness of serous layer fluid. These observations are in line with analytical results of [4,5,8].

5. Conclusion

This paper describes a two-layer unsteady state planar model to estimate mucus transport in the lung airways by addressing mucus as a visco-elastic fluid. It has been observed that the mucus transport rate increases as the shear stress induced by air-motion, the pressure gradient and porosity parameter increase. It also includes that mucus transport rate decreases as its elastic modulus or density increases. Also, it includes that mucus transport rate decreases when viscosity of mucus or serous layer fluid and thickness of serous fluid increase. It is also seen that the mucus transport rate decreases with increase in its transport duration.

References

[1] Agarwal, M. and Verma, V.S. (1998). A viscoelastic two-layer planar unsteady state model for mucus transport due to coughing, Ganita, **49(1)**, 7-18.

[2] Blake, J.R. (1975). On the movement of mucus in the lung, J. Biomechanics, **8**, 179-190.

[3] Hanliang, G., Kanso, E. (2017). A computational study of Mucociliary transport in healthy and diseased environments. European Journal of Computational Mechanics **26**, No. 1 & 2, 4-30 .

[4] King, M. Agarwal, M. and Shukla, J.B. (1993). A planar model for muco-ciliary transport: Effect of mucus visco-elasticity, Biorheology, **30**, 49-61.

[5] Verma, V. S. and Tripathee, S. M. (2013). A planar model for muco-ciliary transport in the human lung: Effects of mucus visco-elasticity, cilia beating and porosity, IJMRS's *International Journal of Mathematical Modeling and Physical Sciences*, **1(1)**, 19-25.

[6] Verma, V.S. and Rana. V. (2015). Mucus transport in the human lungs: A mathematical analysis, Journal of Rajasthan Academy of Physical Sciences, **14(2),** 145-156.

[7] Verma, V. S. (2008). A planar two-layer unsteady state model for mucociliary transport. *J.T.S.,India* **2**, 115–127 .

[8] Verma, V. S. (2009). Mucus transport: A fluid mechanical study. *Journal of Rajasthan Academy of Physical Sciences*, **8**, 371–384.