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Parameter Estimation of Exponentiated Length Biased Exponential Distribution via Bayesian Approach

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Abstract

In this paper, the exponentiated length-biased exponential distribution is considered for Bayesian analysis. The expressions for Bayes estimators of the parameter have been derived under squared error, precautionary, entropy, K-loss, and Al-Bayyati's loss functions by using quasi and gamma priors.

Keywords: Bayesian method, exponentiated length biased exponential distribution, quasi and gamma priors, squared error, precautionary, entropy, K-loss, and Al-Bayyati's loss functions.

Mathematics Subject Classification: 62C10

1. Introduction

Maxwell et al. [1] proposed a new generalization of the length-biased exponential distribution called the exponentiated length-biased exponential (E-LBE) distribution for modeling lifetime data due to some interesting properties such as "lack of memory". The probability density function of E-LBE distribution is given by

$$f\left(x;\theta\right)=\frac{b}{a^2}\theta x e^{-x/a}\left[\left(1+\left(x/a\right)\right)e^{-x/a}\right]^{b-1}\left[1-\left\{\left(1+\left(x/a\right)\right)e^{-x/a}\right\}^{b}\right]^{\theta-1} \quad ; x>0. \tag{1}$$

The joint density function or likelihood function of (1) is given by

$$f\left(\underline{x};\theta\right)=\left(\frac{b}{a^2}\right)^n\theta^n\left(\prod_{i=1}^{n}x_i e^{-x_i/a}\left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right)$$
$$\times exp\left\{-\left(\theta-1\right)\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right\} \tag{2}$$

The log-likelihood function is given by

$$log\ f\left(\underline{x};\theta\right)=n\,log\left(\frac{b}{a^2}\right)+n\,log\,\theta+log\left(\prod_{i=1}^{n}x_i e^{-x_i/a}\left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right)$$

$$-\left(\theta-1\right)\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1} \tag{3}$$

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Differentiating (3) with respect to θ and equating to zero, we get the maximum likelihood estimator of θ, which is given as

$$\hat{\theta} = n \Big/ \sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}. \tag{4}$$

2. Bayesian Method of Estimation

The Bayesian inference procedures have been developed generally under squared error loss function

$$L\left(\hat{\theta},\theta\right)=\left(\hat{\theta}-\theta\right)^2. \tag{5}$$

The Bayes estimator under the above loss function, say, $\hat{\theta}_s$ is the posterior mean, i.e.,

$$\hat{\theta}_S = E\left(\theta\right). \tag{6}$$

Zellner [2], Basu and Ebrahimi [3] have recognized the inappropriateness of using the symmetric loss function. Norstrom [4] introduced precautionary loss function is given as

$$L\left(\hat{\theta},\theta\right)=\frac{\left(\hat{\theta}-\theta\right)^2}{\hat{\theta}}. \tag{7}$$

The Bayes estimator under this loss function is denoted by $\hat{\theta}_P$ and is obtained as

$$\hat{\theta}_P = \left[E\left(\theta^2\right)\right]^{1/2}. \tag{8}$$

Calabria and Pulcini [5] point out that a useful asymmetric loss function is the entropy loss

$$L\left(\Delta\right)\propto\left[\Delta^p - p\ \ log_e\left(\Delta\right)-1\right]$$

where $\Delta=\frac{\hat{\theta}}{\theta}$, and whose minimum occurs at $\hat{\theta}=\theta.$ Also, the loss function $L\left(\Delta\right)$ has been used in Dey et al. [6] and Dey and Liu [7], in the original form having $p=1.$ Thus $L\left(\Delta\right)$ can written be as

$$L\left(\Delta\right)=b\left[\Delta-log_e\left(\Delta\right)-1\right];\ b>0. \tag{9}$$

The Bayes estimator under the entropy loss function is denoted by $\hat{\theta}_E$ and is obtained by solving the following equation

$$\hat{\theta}_E = \left[E\left(\frac{1}{\theta}\right)\right]^{-1}. \tag{10}$$

Wasan [8] proposed the K-loss function, which is given as

$$L\left(\hat{\theta},\theta\right)=\frac{\left(\hat{\theta}-\theta\right)^2}{\hat{\theta}\theta} \tag{11}$$

Under the K-loss function, the Bayes estimator of θ is denoted by $\hat{\theta}_K$ and is obtained as

$$\hat{\theta}_K = \left[\frac{E(\theta)}{E(1/\theta)}\right]^{\frac{1}{2}} \tag{12}$$

Al-Bayyati [9] introduced a new loss function, which is given as

$$L\left(\hat{\theta},\theta\right) = \theta^c \left(\hat{\theta} - \theta\right)^2 \tag{13}$$

Under Al-Bayyati's loss function, the Bayes estimator of θ is denoted by $\hat{\theta}_{Al}$ and is obtained as

$$\hat{\theta}_{Al} = \frac{E\left(\theta^{c+1}\right)}{E\left(\theta^c\right)} \tag{14}$$

Let us consider two prior distributions of θ to obtain the Bayes estimators.

(i) **Quasi-prior:** For the situation where we have no prior information about the parameter θ, we may use the quasi-density as given by

$$g_1(\theta) = \frac{1}{\theta^d} \ ; \theta > 0, \ d \geq 0, \tag{15}$$

where $d = 0$ leads to a diffuse prior and $d = 1$, a non-informative prior.

(ii) **Gamma prior:** Generally, the gamma density is used as the prior distribution of the parameter θ given by

$$g_2(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} \ ; \theta > 0. \tag{16}$$

3. Posterior density under $g_1(\theta)$

The posterior density of θ under $g_1(\theta)$, on using (2), is given by

$$f(\theta/\underline{x}) = \frac{\left[\begin{array}{l}\left(\frac{b}{a^2}\right)^n \theta^n \left(\prod_{i=1}^{n} x_i\, e^{-x_i/a} \left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right) \\ \times exp\left\{-(\theta-1)\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right\}\theta^{-d}\end{array}\right]}{\int_0^\infty \left[\begin{array}{l}\left(\frac{b}{a^2}\right)^n \theta^n \left(\prod_{i=1}^{n} x_i\, e^{-x_i/a} \left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right) \\ \times exp\left\{-(\theta-1)\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right\}\theta^{-d}\end{array}\right] d\theta}$$

$$= \frac{\theta^{n-d} \ exp\left[-\theta\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right]}{\int_0^\infty \theta^{n-d} \ exp\left[-\theta\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right] d\theta}$$

$$= \frac{\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n-d+1}}{\Gamma\left(n-d+1\right)} \theta^{n-d} e^{-\theta\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}} \quad (17)$$

Theorem 1: On using (17), we have

$$E\left(\theta^{c}\right)=\frac{\Gamma\left(n-d+c+1\right)}{\Gamma\left(n-d+1\right)}\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-c} \quad (18)$$

Proof: By definition,

$$E\left(\theta^{c}\right)=\int \theta^{c} f\left(\theta/\underline{x}\right)d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n-d+1}}{\Gamma\left(n-d+1\right)} \int_{0}^{\infty} \theta^{(n-d+c)} e^{-\theta\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}} d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n-d+1}}{\Gamma\left(n-d+1\right)} \times \frac{\Gamma\left(n-d+c+1\right)}{\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n-d+c+1}}$$

$$= \frac{\Gamma\left(n-d+c+1\right)}{\Gamma\left(n-d+1\right)}\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-c}.$$

From equation (18), for $c=1$, we have

$$E\left(\theta\right)=\left(n-d+1\right)\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1} \quad (19)$$

From equation (18), for $c=2$, we have

$$E\left(\theta^{2}\right)=\left[\left(n-d+2\right)\left(n-d+1\right)\right]\left[\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right]^{-2} \quad (20)$$

From equation (18), for $c=-1$, we have

$$E\left(\frac{1}{\theta}\right)=\frac{1}{\left(n-d\right)}\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1} \quad (21)$$

From equation (18), for $c=c+1$, we have

$$E\left(\theta^{c+1}\right)=\frac{\Gamma\left(n-d+c+2\right)}{\Gamma\left(n-d+1\right)}\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-(c+1)} \quad (22)$$

4. Bayes estimators under $g_1(\theta)$

From equation (6), on using (19), the Bayes estimator of θ under the squared error loss function is given by

$$\hat{\theta}_S = (n-d+1)\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)^{-1} \tag{23}$$

From equation (8), on using (20), the Bayes estimator of θ under the precautionary loss function is obtained as

$$\hat{\theta}_P = \left[(n-d+2)(n-d+1)\right]^{\frac{1}{2}}\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)^{-1} \tag{24}$$

From equation (10), on using (21), the Bayes estimator of θ under the entropy loss function is given by

$$\hat{\theta}_E = (n-d)\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)^{-1} \tag{25}$$

From equation (12), on using (19) and (21), the Bayes estimator of θ under the K-loss function is given by

$$\hat{\theta}_K = \left[(n-d+1)(n-d)\right]^{\frac{1}{2}}\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)^{-1} \tag{26}$$

From equation (14), on using (18) and (22), the Bayes estimator of θ under Al-Bayyati's loss function comes out to be

$$\hat{\theta}_{Al} = (n-d+c+1)\left(\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)^{-1} \tag{27}$$

5. Posterior density under $g_2(\theta)$

Under $g_2(\theta)$, the posterior density of θ, using equation (2), is obtained as

$$f(\theta/\underline{x}) = \frac{\left[\begin{array}{l}\left(\frac{b}{a^2}\right)^n \theta^n \left(\prod_{i=1}^{n} x_i\, e^{-x_i/a}\left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right) \\ \times exp\left\{-(\theta-1)\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right\}\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}\end{array}\right]}{\int_0^\infty \left[\begin{array}{l}\left(\frac{b}{a^2}\right)^n \theta^n \left(\prod_{i=1}^{n} x_i\, e^{-x_i/a}\left[\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right]^{b-1}\right) \\ \times exp\left\{-(\theta-1)\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right\}\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}\end{array}\right] d\theta}$$

$$= \frac{\theta^{n+\alpha-1}\ exp\left[-\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)\theta\right]}{\int_0^\infty \theta^{n+\alpha-1}\ exp\left[-\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^b\right]^{-1}\right)\theta\right] d\theta}$$

$$= \frac{\theta^{n+\alpha-1}\ exp\left[-\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)\theta\right]}{\Gamma\left(n+\alpha\right)\Big/\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n+\alpha}}$$

$$= \frac{\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n+\alpha}}{\Gamma\left(n+\alpha\right)}\ \theta^{n+\alpha-1}\, e^{-\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)\theta} \tag{28}$$

Theorem 2: On using (28), we have

$$E\left(\theta^{c}\right)=\frac{\Gamma\left(n+\alpha+c\right)}{\Gamma\left(n+\alpha\right)}\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-c} \tag{29}$$

Proof: By definition,

$$E\left(\theta^{c}\right)=\int\theta^{c} f\left(\theta/\underline{x}\right)d\theta$$

$$= \frac{\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n+\alpha}}{\Gamma\left(n+\alpha\right)}\int_{0}^{\infty}\theta^{n+\alpha+c-1}\ e^{-\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)\theta} d\theta$$

$$= \frac{\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n+\alpha}}{\Gamma\left(n+\alpha\right)}\frac{\Gamma\left(n+\alpha+c\right)}{\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{n+\alpha+c}}$$

$$= \frac{\Gamma\left(n+\alpha+c\right)}{\Gamma\left(n+\alpha\right)}\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-c}.$$

From equation (29), for $c=1$, we have

$$E\left(\theta\right)=\left(n+\alpha\right)\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1} \tag{30}$$

From equation (29), for $c=2$, we have

$$E\left(\theta^{2}\right)=\left[\left(n+\alpha+1\right)\left(n+\alpha\right)\right]\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-2} \tag{31}$$

From equation (29), for $c=-1$, we have

$$E\left(\frac{1}{\theta}\right)=\frac{1}{\left(n+\alpha-1\right)}\left(\beta+\sum_{i=1}^{n} log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right) \tag{32}$$

From equation (29), for $c=c+1$, we have

$$E\left(\theta^{c+1}\right)=\frac{\Gamma(n+\alpha+c+1)}{\Gamma(n+\alpha)}\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-(c+1)}. \tag{33}$$

6. Bayes estimators under $g_2(\theta)$

From equation (6), on using (30), the Bayes estimator of θ under the squared error loss function is given by

$$\hat{\theta}_S=(n+\alpha)\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1}. \tag{34}$$

From equation (8), on using (31), the Bayes estimator of θ under the precautionary loss function is obtained as

$$\hat{\theta}_P=\left[(n+\alpha+1)(n+\alpha)\right]^{\frac{1}{2}}\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1}. \tag{35}$$

From equation (10), on using (32), the Bayes estimator of θ under the entropy loss function is given by

$$\hat{\theta}_E=(n+\alpha+1)\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1}. \tag{36}$$

From equation (12), on using (30) and (32), the Bayes estimator of θ under the K-loss function is given by

$$\hat{\theta}_K=\left[(n+\alpha)(n+\alpha-1)\right]^{\frac{1}{2}}\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1}. \tag{37}$$

From equation (14), on using (29) and (33), the Bayes estimator of θ under Al-Bayyati's loss function comes out to be

$$\hat{\theta}_{Al}=(n+\alpha+c)\left(\beta+\sum_{i=1}^{n}log\left[1-\left\{\left(1+\left(x_i/a\right)\right)e^{-x_i/a}\right\}^{b}\right]^{-1}\right)^{-1}. \tag{38}$$

7. Conclusion

In this paper, we have obtained a number of estimators of the parameter of exponentiated sslength-biased exponential distribution. In equation (4) we have obtained the maximum likelihood estimator of the parameter. In equations (23), (24), (25), (26) and (27) we have obtained the Bayes estimators under different loss functions using quasi-prior. In equations (34), (35), (36), (37) and (38) we have obtained the Bayes estimators under different loss functions using gamma prior. In the above equations, it is clear that the Bayes estimators depend upon the parameters of the prior distribution.

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